

# Solving the vehicle routing problem with drone for delivery services using an ant colony optimization algorithm

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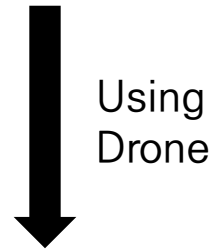
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# 1. Introduction

- Over the past few decades, the logistics industry has experienced rapid development.
- With the growth of the logistics industry, companies face the challenge of efficiently and effectively meeting increasing customer demands.
- To address this issue, new technologies have been introduced.



- The implementation of these new technologies has resulted in cost reductions, shorter delivery times, and improved customer satisfaction.
- Drones have the advantage of reducing delivery delays as they do not face traffic congestion.

# 1. Introduction

## VRP (Vehicle Routing Problem)

- One of the important issues in logistics management.
- Traditional VRP means providing delivery services to customers using trucks located at warehouses.
- However, it is not well-suited for cases where parking conditions are poor or vehicle accessibility is limited.



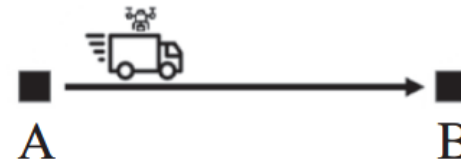
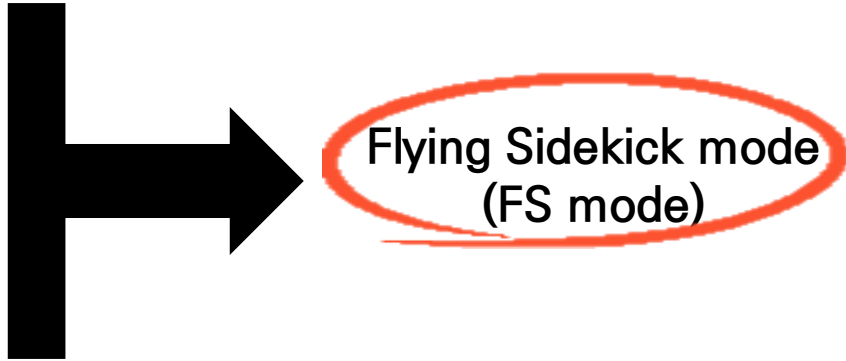
## VRPD (Vehicle Routing Problem with Drone)

- Vehicle Routing Problem (VRP) with the use of drones.
- The VRPD problem is addressed using the ACO algorithm to propose two new methods.
- The research is conducted by dividing it into small-scale instances and large-scale instances.

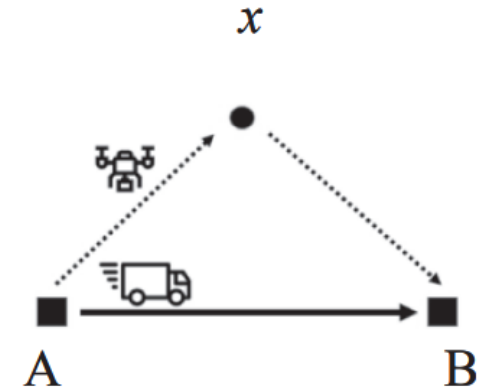
## 2. Problem description

### a) stationary

- When the drone moves from node A to node B while conserving its battery, it means not making another delivery.



a) stationary



b) in-flight

### b) in-flight

- When the truck moves from node A to node B, and the drone makes a delivery to a customer 'x' before moving to node B, it means.

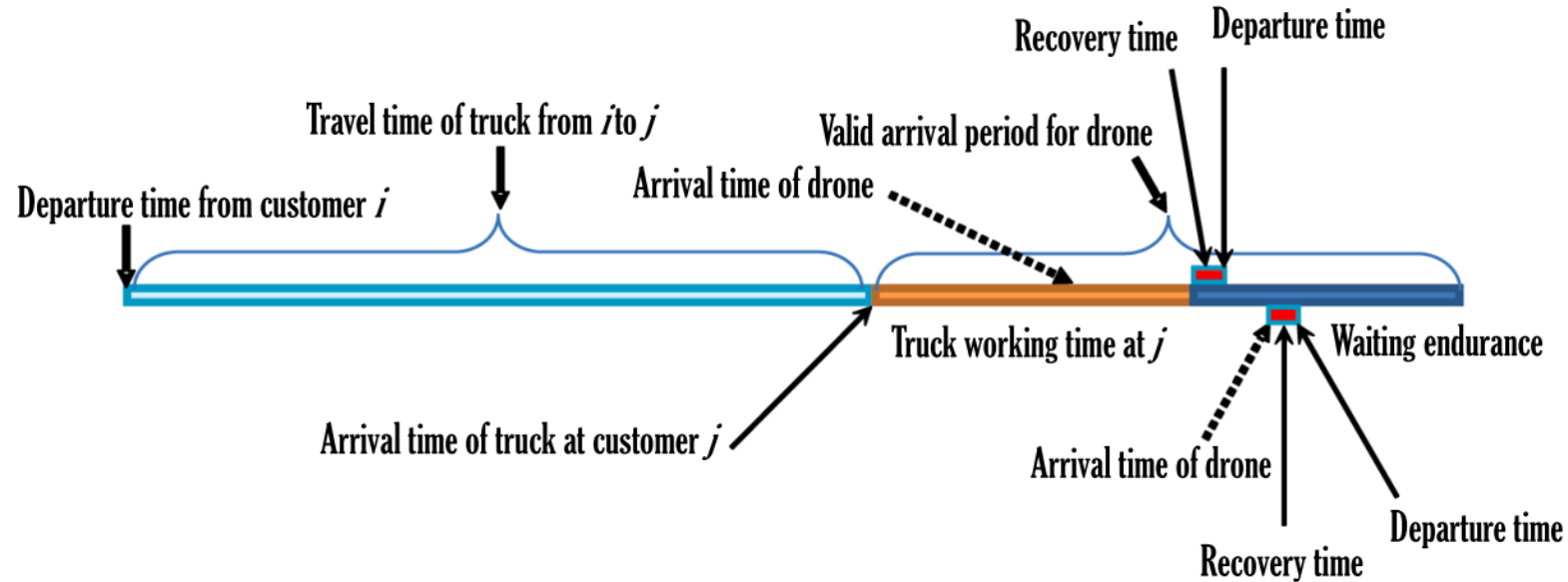
## 2. Problem description

1. All vehicles must start their routes from the depot and end at the depot before the end of the work shift.
2. Both trucks and drones must complete their services within the working hours, and each node can be served by either a truck or a drone, but not both.
3. Trucks have a capacity constraint, while drones can deliver only one item at a time.
4. There is a one-to-one matching between trucks and drones.

## 2. Problem description

5. Drones in stationary mode do not incur travel costs.
6. Drones require a recovery time before departing to the next location, but this recovery time can be neglected due to the one-item delivery.
7. After completing a delivery, the drone must immediately move to the next node of the truck, and the truck should be at the next node before the drone's arrival.

## 2. Problem description



(a) When the truck is servicing or waiting at customer  $j$

- If the drone arrives at location  $j$  **before** the truck:
  - Departure time = Truck's delivery time + Drone's recovery time
- If the drone arrives at location  $j$  **after** the truck:
  - Departure time = Drone's arrival time + Drone's recovery time



# 3. Methodology

## ACO (Ant Colony Optimization) algorithm

- An algorithm inspired by the foraging behavior of ants to solve path-finding problems.

## ACS (Ant Colony System)

- Dorigo and Gambardella proposed it.
- ACS consists of three main procedures: the selection method, local pheromone update, and global pheromone update.
- VRPD is based on ACS.

# 3. Methodology

## *FS* mode

- Delivery is possible for only one customer.

## *FS<sup>+</sup>* mode

- While traveling with the truck, it is possible to make deliveries to customers.
- It is possible to make deliveries to multiple customers, not just one.
- Have the issue of “excess halt”

## *FS<sup>+1</sup>* mode

- “excess halt” has been resolved.

## Excess Halt

- It refers to excessive truck waiting time at one location.

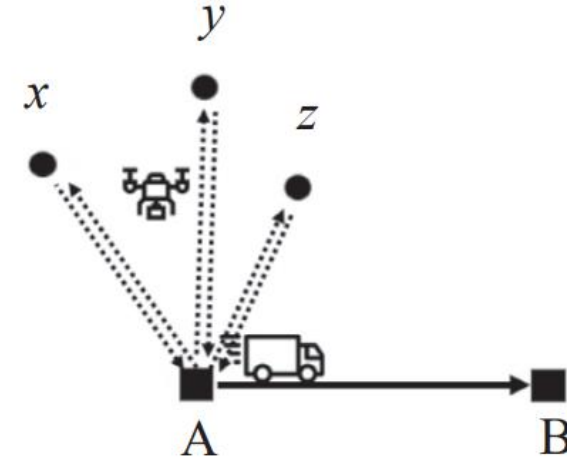


Fig. 3. Additional flying action of the drone (*FS<sup>+</sup>* mode).

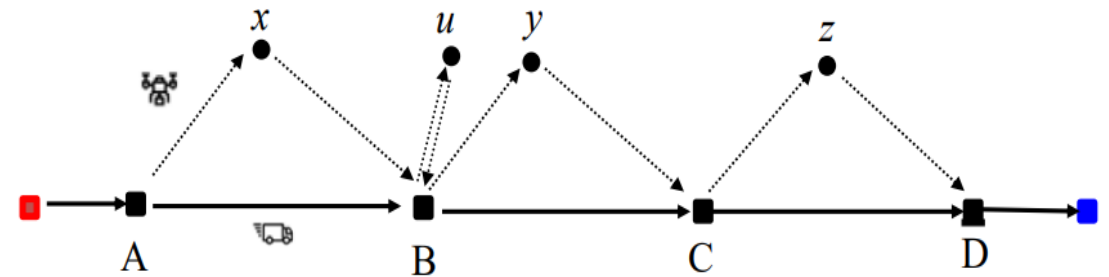


Fig. 4. Example of *FS<sup>+1</sup>* method.

### 3. Methodology

#### *FS* mode

$$\sum_{j \in I: i \neq j} x_{ijk}^D \leq 1 \quad \forall k \in K, i \in N$$

$$w_{ik}^T \leq W y_{ik} \quad \forall k \in K, i \in I$$

#### *FS*<sup>+</sup>, *FS*<sup>+1</sup> mode

$$\sum_{j \in I: i \neq j} x_{ijk}^D \leq M \quad \forall k \in K, i \in N$$

$$w_{ik}^T \leq \pi W y_{ik} \quad \forall k \in K, i \in I$$

- *FS*<sup>+</sup> mode : Adjustment of the number of drones
- *FS*<sup>+1</sup> mode : Control of drone working hours

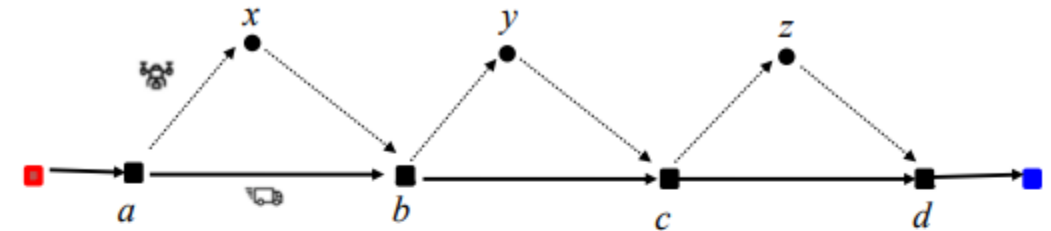
# 3. Methodology

## 2-opt

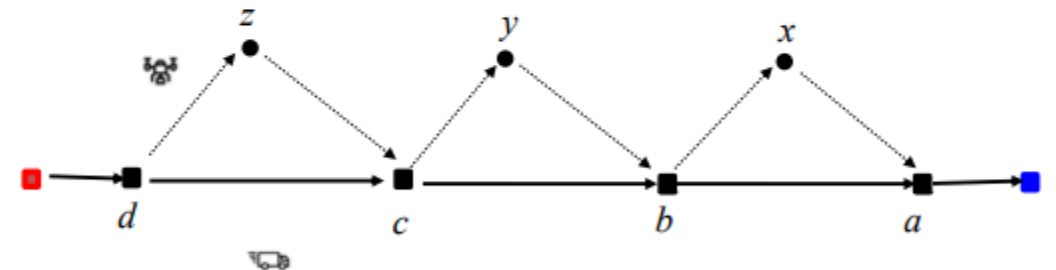
- A simple yet effective local search algorithm used in routing problems.

## After

- The truck now visits customers in the order of d, c, b, a, and the drone provides services in the order of z, y, x.
- The 2-opt method is used to reduce the total distance of the route and to increase delivery efficiency.
- The 2-opt method is repeatedly applied until the best route is obtained.



a) Visit sequence before applying 2-opt method



b) Visit sequence after applying 2-opt method

Fig. 6. Illustration of 2-opt on a truck-drone route.

## 4. Computational tasks

Dataset

Comprised of three subsets with customers located in Clusters (C), Randomly (R), and a combination of Random and Clusters (RC).

Maximum working time of a drone

30 hours

Maximum working time of the truck

480 hours

Maximum truckload capacity

500 units

# 4. Computational tasks

## 4.1 Comparison between ACO and optimal solutions for small instances

**Table 1**

Comparison results between ACO and optimization model for the VRPD (FS mode).

Instances	NC	Model Results				ACO Results			$\Delta\%$ Total Cost
		CPU (s)	SF	Total Cost	GAP Model	CPU (s)	SF	Total Cost	
C01	5	0.33	1	4873.17	0.02%	0.19	1	4873.18	0.00%
	10	140.59	1	4938.54	0.02%	0.48	1	4938.84	0.01%
	15	7200	1	4962.89	1.12%	0.87	1	4963.87	0.02%
	20	7200	1	4978.56	1.61%	1.49	1	4980.38	0.04%
	25	7200	1	4996.33	1.97%	2.2	1	4998.24	0.04%
	30	7200	1	5116.10	3.59%	4.08	1	5135.20	0.37%
R01	5	0.45	1	5030.66	0.10%	0.18	1	5030.66	0.00%
	10	303.05	1	5192.96	0.01%	0.54	1	5192.96	0.00%
	15	7200	1	5261.23	3.56%	0.75	1	5265.69	0.08%
	20	7200	1	5311.98	3.84%	1.44	1	5317.82	0.11%
	25	7200	1	5394.35	5.55%	1.81	1	5419.35	0.46%
	30	7200	1	5440.49	7.45%	3.69	1	5473.95	0.61%
RC01	5	1.03	1	5249.82	0.03%	0.2	1	5249.82	0.00%
	10	126.86	1	5145.35	0.04%	0.48	1	5145.35	0.00%
	15	7200	1	5226.11	2.56%	0.88	1	5226.11	0.00%
	20	7200	1	5189.7	2.63%	1.57	1	5203.46	0.27%
	25	7200	1	5228.99	3.53%	2.04	1	5281.89	1.01%
	30	7200	1	5270.3	5.10%	3.91	1	5325.76	1.05%

*Note:* NC: number of customers; CPU: CPU time (s); SF: number of sub-fleets; Total: total cost; GAP Model: Integrality GAP;  $\Delta\%$  Total Cost: ratio between the ACO and model cost difference and the model cost.

# 4. Computational tasks

## 4.2 ACO results with large instances

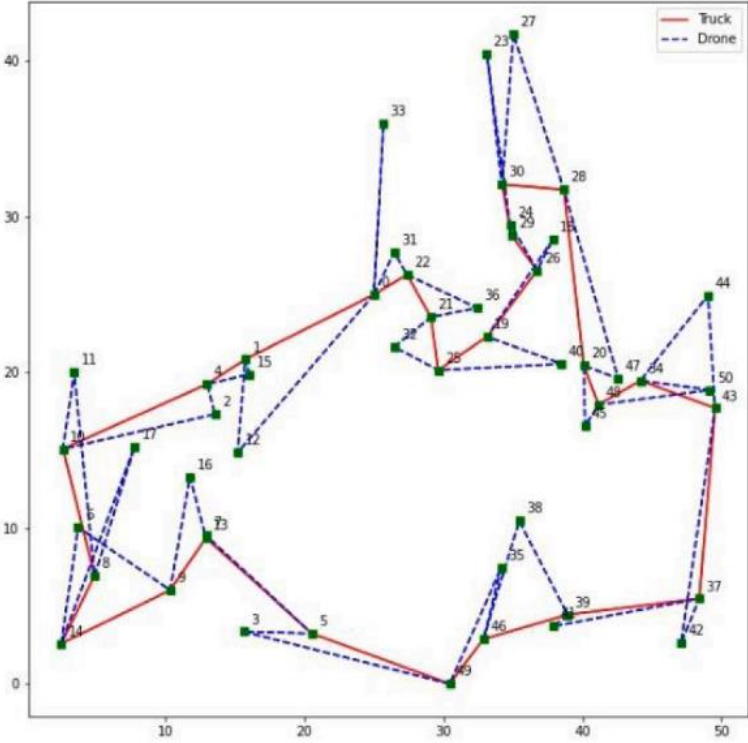
**Table 2**  
Best output results for the benchmark using the ACO (FS mode).

Instances	Parameters				SF	CPU (s)	Distances		Costs			
	NC	$\alpha$	$\beta$	z			TTD	DTD	TFC	TTC	DTC	Total Cost
C01	50	2	8	0	1	11.09	121.91	221.29	4796	402.30	73.03	5271.32
C02	50	4	9	0.05	2*	8.51	156.64	353.66	5232	516.91	116.71	5865.63
C03	50	9	6	0.2	2	7.62	180.64	251.80	9592	596.11	83.09	10271.19
C04	50	9	6	0	2*	8.68	149.55	279.05	5232	493.52	92.09	5817.60
C05	100	9	8	0	2	29.82	158.47	328.33	9592	522.95	108.35	10223.31
C06	100	7	8	0	2	32.66	175.19	352.00	9592	578.13	116.16	10286.30
C07	100	3	7	0.15	2	29.31	216.88	374.66	9592	715.70	123.64	10431.35
C08	100	9	5	0	2	26.22	214.46	400.73	9592	707.72	132.24	10431.97
C09	200	2	9	0.05	4	138.66	362.31	682.71	19,184	1195.62	225.29	20604.90
C10	200	4	9	0	4	126.92	317.99	542.51	19,184	1049.37	179.03	20412.40
C11	200	3	8	0.25	4	137.26	325.78	486.29	19,184	1075.07	160.48	20419.55
C12	200	7	8	0.15	4	133.80	357.47	534.03	19,184	1179.65	176.23	20539.88
R01	50	10	10	0.1	2	9.3	231.56	355.86	9592	764.15	117.43	10473.60
R02	50	5	7	0.25	2	9.31	200.68	374.52	9592	662.24	123.59	10377.84
R03	50	5	7	0.1	2	8.25	205.54	399.46	9592	678.28	131.82	10402.10
R04	50	3	6	0.1	2	7.33	220.66	361.61	9592	728.18	119.33	10439.50
R05	100	6	8	0.3	3	30.58	330.13	552.75	14,388	1089.43	182.41	15659.83
R06	100	6	6	0.1	3	28.06	326.16	700.00	14,388	1076.33	231.00	15695.33
R07	100	3	8	0	3	31.41	332.95	655.63	14,388	1098.74	216.36	15703.10
R08	100	6	8	0.25	3	32.44	329.27	503.72	14,388	1086.59	166.23	15640.82
R09	200	2	10	0	4	137.42	482.82	936.31	19,184	1593.31	308.98	21086.26
R10	200	5	9	0	4	139.03	485.60	954.93	19,184	1602.48	315.13	21101.61
R11	200	1	10	0	4	137.35	459.95	922.34	19,184	1517.84	304.37	21006.22
R12	200	7	8	0	4	137.76	480.24	941.83	19,184	1584.79	310.80	21079.60
RC01	50	2	6	0.05	2	7.89	181.76	360.71	9592	599.81	119.03	10310.85
RC02	50	8	9	0.15	2	9.47	215.94	346.96	9592	712.60	114.50	10419.11
RC03	50	10	8	0.3	2	8.79	179.97	338.89	9592	593.90	111.83	10297.75
RC04	50	10	10	0	2*	8.94	153.39	297.89	5232	506.19	98.30	5836.50
RC05	100	7	8	0	3	31.63	261.95	508.14	14,388	864.44	167.69	15420.14
RC06	100	9	10	0.05	2	31.19	291.66	527.97	9592	962.48	174.23	10728.72
RC07	100	9	9	0.05	2	29.93	262.90	588.30	9592	867.57	194.14	10653.71
RC08	100	1	10	0.15	3	31.24	275.81	458.56	14,388	910.17	151.32	15449.51
RC09	200	1	10	0	4	138.26	463.83	839.87	19,184	1530.64	277.16	20991.80
RC10	200	5	10	0.1	4	141.89	445.06	720.07	19,184	1468.70	237.62	20890.32
RC11	200	2	9	0	4	136.42	420.40	804.36	19,184	1387.32	265.44	20836.77
RC12	200	8	10	0	4	138.95	423.00	790.61	19,184	1395.90	260.90	20840.82

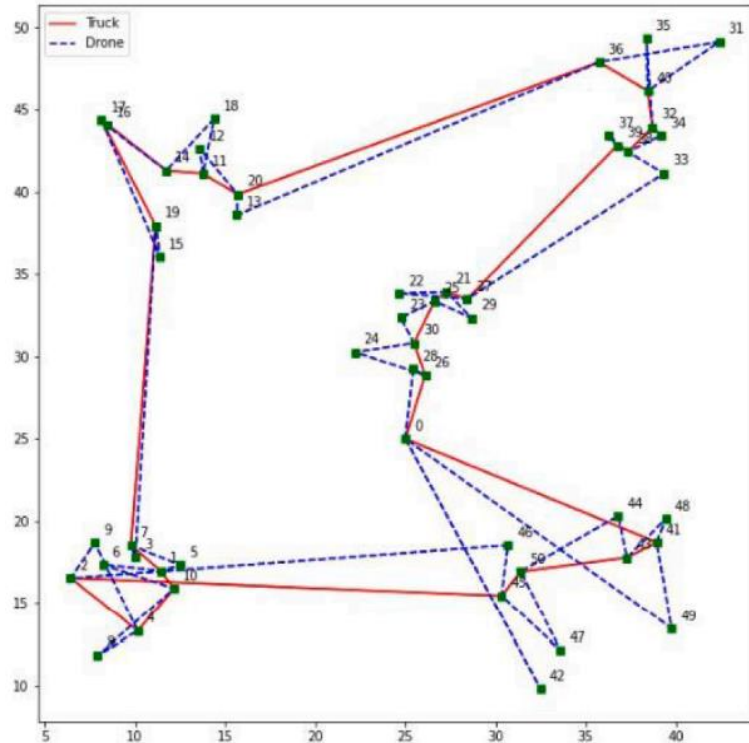
Note: \*: one drone is dispatched as 2nd sub-fleet; NC: number of customers; SF: number of sub-fleets; CPU: CPU time (s);  $\alpha$ : pheromone intensity weight in Eq. (34);  $\beta$ : pheromone visibility weight in Eq. (34); z: parameter for determining a drone's action; TTD: truck travel distance; DTD: drone travel distance; TFC: total fixed costs; TTC: truck travel costs; DTC: drone travel costs.

# 4. Computational tasks

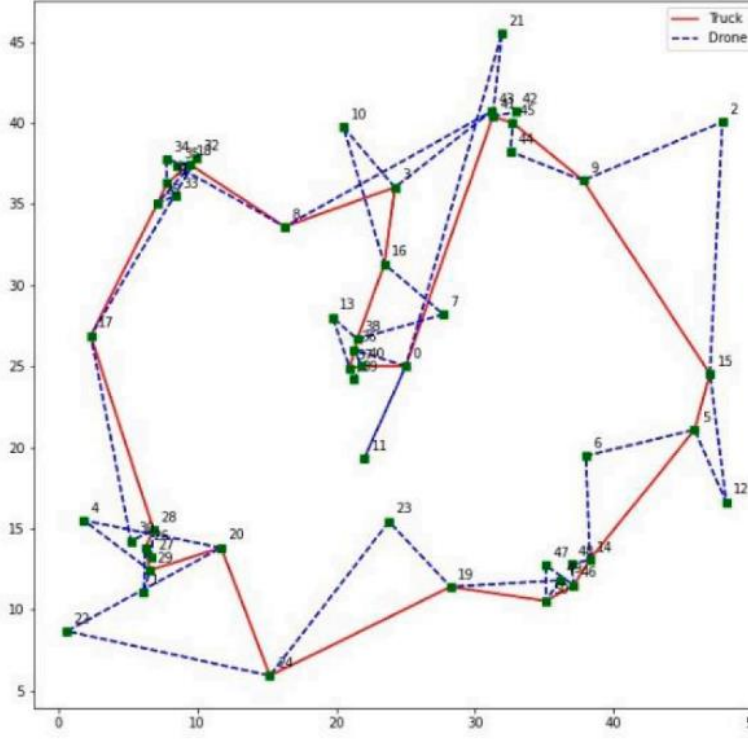
## 4.2 ACO results with large instances



a) Instance C02



b) Instance C04



c) Instance RC04



# 4. Computational tasks

## 4.3 Comparison results using $FS$ , $FS^+$ , and $FS^{+1}$ modes

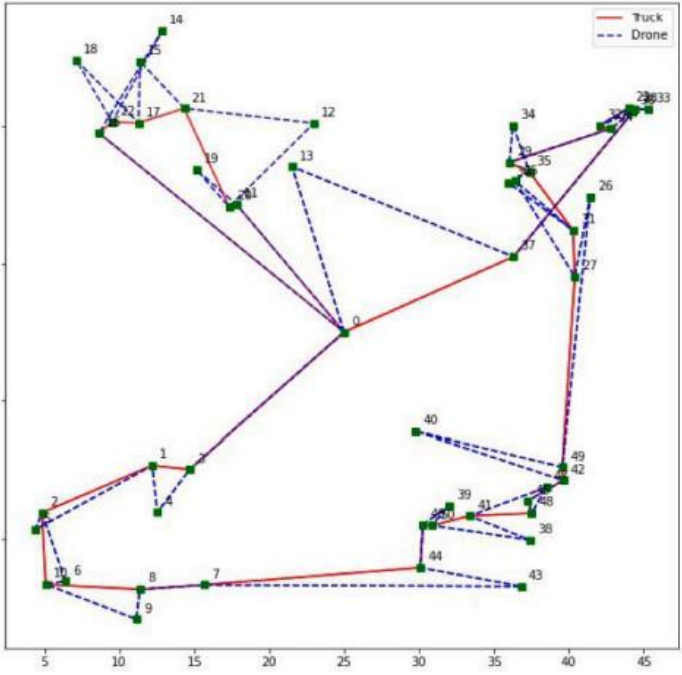
**Table 3**  
Output results of VRPD for FS,  $FS^+$ , and  $FS^{+1}$  methods.

Instances	FS		$FS^+$		$FS^{+1}$			$\Delta\%Total\ Cost$			
	SF	Total Cost	$\pi$	SF	Total Cost	$\pi$	SF	Total Cost	$\gamma_1$	$\gamma_2$	$\gamma_3$
C01	1	5271.32	0.3	1	5273.96	0.3	1	<b>5263.29</b>	0.05%	-0.15%	-0.20%
C02	2*	5865.63	0.2	2*	<b>5822.58</b>	0.1	2*	5894.99	-0.73%	0.50%	1.24%
C03	2	10271.19	0.8	2	<b>10171.39</b>	1	2	10254.77	-0.97%	-0.16%	0.82%
C04	2*	<b>5817.6</b>	1	2*	5824.24	0.7	2	10169.69	0.11%	74.81%	74.61%
C05	2	10223.31	0.3	2	10190.97	0.5	2	<b>10178.78</b>	-0.32%	-0.44%	-0.12%
C06	2	10286.3	0.2	2	10296.50	0.4	2	<b>10284.54</b>	0.10%	-0.02%	-0.12%
C07	2	10431.35	0.3	2	10409.12	0.3	2	<b>10406.04</b>	-0.21%	-0.24%	-0.03%
C08	2	<b>10431.97</b>	0.1	2	10433.25	0.1	2	10450.14	0.01%	0.17%	0.16%
C09	4	20604.9	0.1	4	20648.85	0.3	4	<b>20601.45</b>	0.21%	-0.02%	-0.23%
C10	4	20412.4	1	9**	<b>18668.31</b>	0.7	4	20364.22	-8.54%	-0.24%	9.08%
C11	4	20419.55	0.2	4	20424.10	0.5	4	<b>20340.55</b>	0.02%	-0.39%	-0.41%
C12	4	20539.88	0.1	4	20591.51	0.1	4	<b>20524.22</b>	0.25%	-0.08%	-0.33%
R01	2	10473.6	0.7	2	<b>10383.55</b>	0.7	2	10427.67	-0.86%	-0.44%	0.42%
R02	2	10377.84	0.8	2	<b>10351.50</b>	1	2	10372.25	-0.25%	-0.05%	0.20%
R03	2	10402.1	0.8	2	10363.59	1	2	<b>10356.25</b>	-0.37%	-0.44%	-0.07%
R04	2	10439.5	0.8	2	<b>10332.77</b>	1	2	10384.62	-1.02%	-0.53%	0.50%
R05	3	15659.83	0.5	3	<b>15597.47</b>	0.9	3	15618.67	-0.40%	-0.26%	0.14%
R06	3	15695.33	0.5	3	15624.17	0.7	3	<b>15618.71</b>	-0.45%	-0.49%	-0.03%
R07	3	15703.1	0.6	3	<b>15585.10</b>	0.6	3	15602.87	-0.75%	-0.64%	0.11%
R08	3	15640.82	0.5	3	15561.98	0.8	3	<b>15556.00</b>	-0.50%	-0.54%	-0.04%
R09	4	21086.26	0.1	4	<b>21004.52</b>	0.1	4	21108.37	-0.39%	0.10%	0.49%
R10	4	21101.61	0.2	4	<b>21005.23</b>	0.1	4	21099.53	-0.46%	-0.01%	0.45%
R11	4	<b>21006.22</b>	0.1	4	21069.37	0.1	4	21089.72	0.30%	0.40%	0.10%
R12	4	<b>21079.6</b>	0.1	4	21090.62	0.1	4	21100.88	0.05%	0.10%	0.05%
RC01	2	10310.85	0.3	2	10296.97	1	2	<b>10266.83</b>	-0.13%	-0.43%	-0.29%
RC02	2	10419.11	0.9	2	<b>10323.85</b>	1	2	10378.27	-0.91%	-0.39%	0.53%
RC03	2	10297.75	0.7	2	10250.24	0.8	2	<b>10223.54</b>	-0.46%	-0.72%	-0.26%
RC04	2*	<b>5836.5</b>	0.8	2	10200.24	0.1	2*	5852.17	74.77%	0.27%	-42.63%
RC05	3	<b>15420.14</b>	0.4	3	15454.60	0.9	3	15434.72	0.22%	0.09%	-0.13%
RC06	2	<b>10728.72</b>	0.1	3*	11145.33	0.1	2	10765.53	3.88%	0.34%	-3.41%
RC07	2	<b>10653.71</b>	0.6	3	15460.19	0.1	2	10723.67	45.12%	0.66%	-30.64%
RC08	3	15449.51	0.4	3	15418.75	1	3	<b>15392.56</b>	-0.20%	-0.37%	-0.17%
RC09	4	<b>20991.8</b>	0.2	4	21010.57	0.2	4	21085.41	0.09%	0.45%	0.36%
RC10	4	20890.32	0.2	4	20909.28	0.2	4	<b>20878.90</b>	0.09%	-0.05%	-0.15%
RC11	4	<b>20836.77</b>	0.1	4	20866.71	0.1	4	20840.60	0.14%	0.02%	-0.13%
RC12	4	20840.82	0.2	4	<b>20832.99</b>	0.3	4	20859.05	-0.04%	0.09%	0.13%

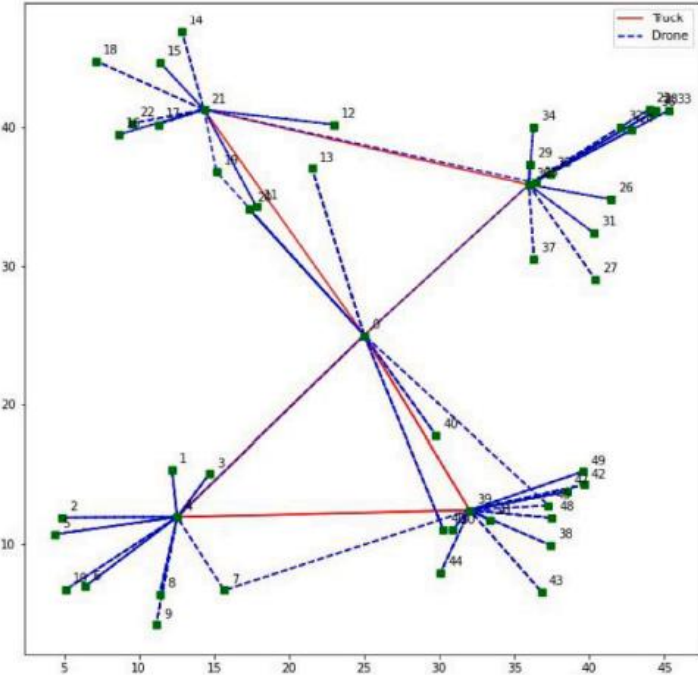
Note: \*: Only one drone of the 2nd sub-fleet is dispatched; \*\*: only one drone from the 4th to the 9th sub-fleet are dispatched;  $\gamma_1$ :  $(FS^+ \text{ Total Cost} - FS \text{ Total Cost}) / (FS^+ \text{ Total Cost})$ ;  $\gamma_2$ :  $(FS^{+1} \text{ Total Cost} - FS \text{ Total Cost}) / (FS \text{ Total Cost})$ ;  $\gamma_3$ :  $(FS^{+1} \text{ Total cost} - FS^+ \text{ Total Cost}) / (FS^+ \text{ Total Cost})$ .

# 4. Computational tasks

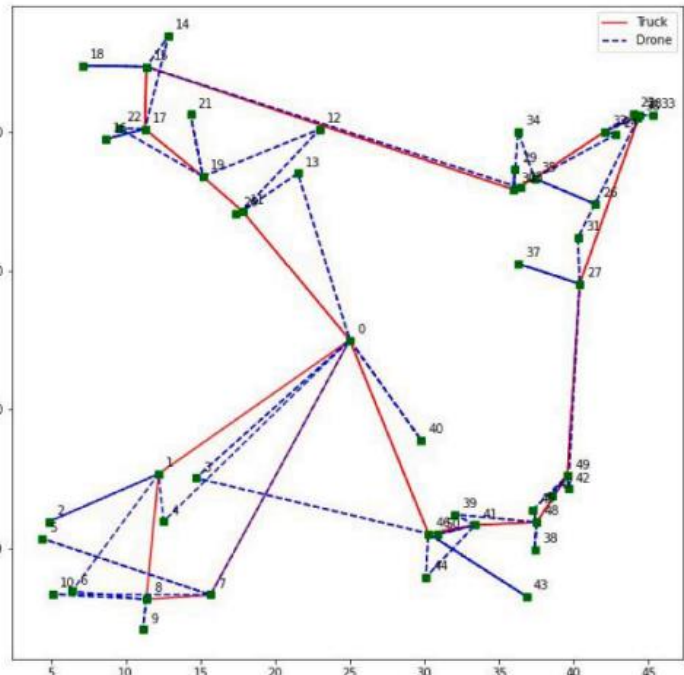
## 4.3 Comparison results using $FS$ , $FS^+$ , and $FS^{+1}$ modes



(a) FS Mode (Total Cost: 10,271.19)



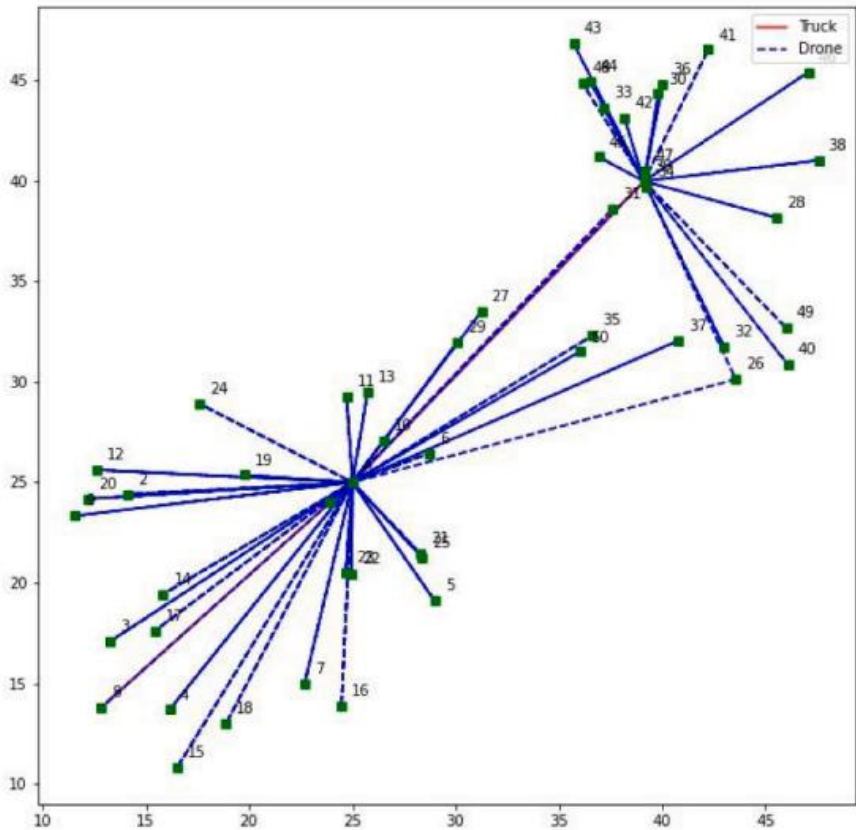
(b) FS+ Mode (Total Cost: 10,171.39)



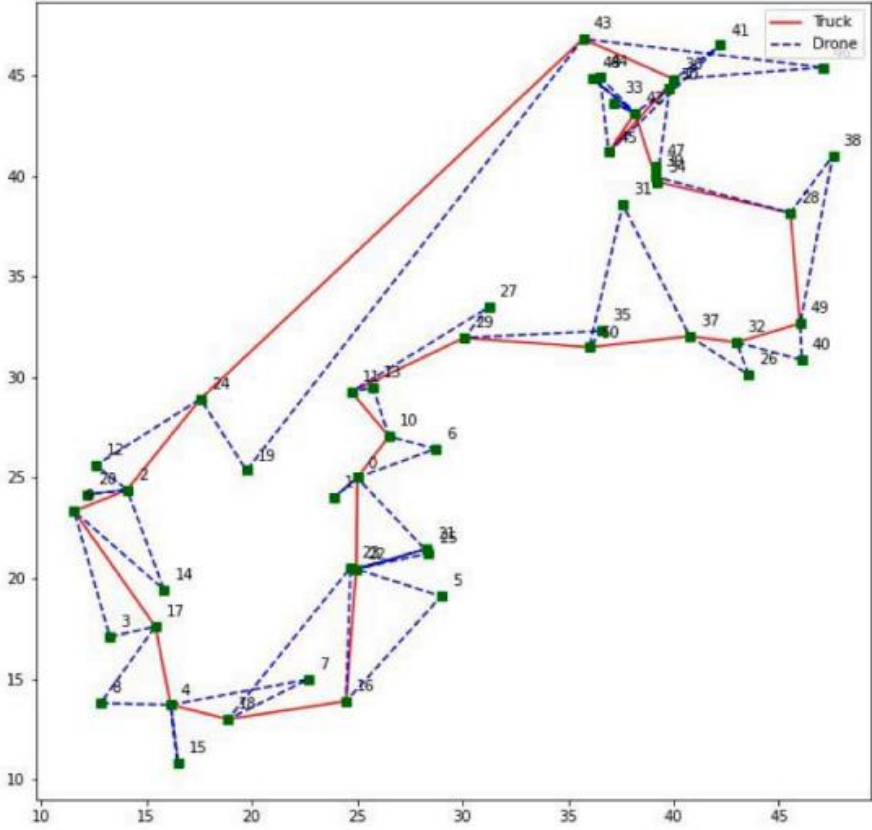
(c) FS+1 Mode (Total Cost: 10,254.77)

# 4. Computational tasks

## 4.3 Comparison results using $FS$ , $FS^+$ , and $FS^{+1}$ modes



a)  $\pi=1$



b)  $\pi=0.3$

# 4. Computational tasks

## 4.4 Comparison of VRPD with VRP

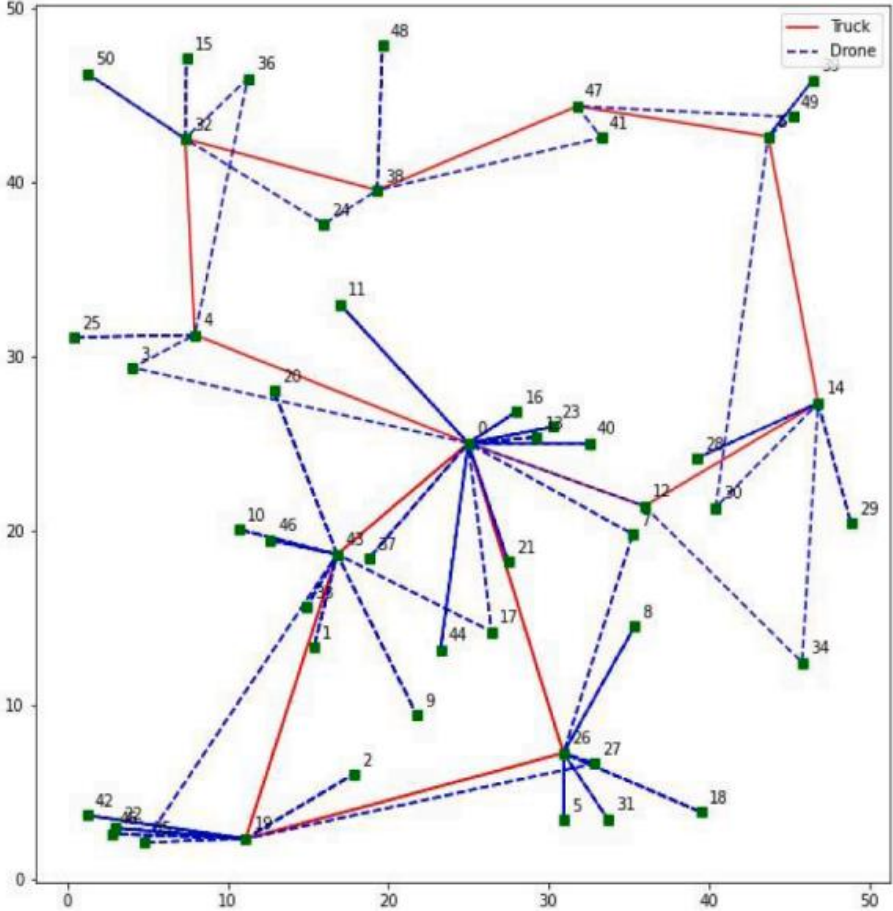
**Table 4**  
Results of all instances for VRPD and VRP (only trucks).

Instance	NC	VRPD						VRP		Total Cost Differences		
		FS mode		FS <sup>+</sup> mode		FS <sup>+</sup> mode		NT	Total Cost (\$)	$\Delta_1$	$\Delta_2$	$\Delta_3$
		SF	Total Cost (\$)	SF	Total Cost (\$)	SF	Total Cost (\$)					
C01	50	1	5271.32	1	5273.96	1	5263.29	2	9266.531	43.1%	43.1%	43.2%
C02	50	2*	5865.63	2*	5822.58	2*	5894.99	2	9611.51	39.0%	39.4%	38.7%
C03	50	2	10271.19	2	10171.39	2	10254.77	2	9502.65	-8.1%	-7.0%	-7.9%
C04	50	2*	5817.60	2*	5824.24	2	10169.69	2	9437.06	38.4%	38.3%	-7.8%
C05	100	2	10223.31	2	10190.97	2	10178.78	3	13887.07	26.4%	26.6%	26.7%
C06	100	2	10286.30	2	10296.50	2	10284.54	4	18431.90	44.2%	44.1%	44.2%
C07	100	2	10431.35	2	10409.12	2	10406.04	4	18580.04	43.9%	44.0%	44.0%
C08	100	2	10431.97	2	10433.25	2	10450.14	4	18592.18	43.9%	43.9%	43.8%
C09	200	4	20604.90	4	20648.85	4	20601.45	7	32469.90	36.5%	36.4%	36.6%
C10	200	4	20412.40	9**	18668.31	4	20364.22	6	27645.41	26.2%	32.5%	26.3%
C11	200	4	20419.55	4	20424.10	4	20340.55	6	27790.61	26.5%	26.5%	26.8%
C12	200	4	20539.88	4	20591.51	4	20524.22	7	32240.10	36.3%	36.1%	36.3%
R01	50	2	10473.60	2	10383.55	2	10427.67	3	14175.99	26.1%	26.8%	26.4%
R02	50	2	10377.84	2	10351.50	2	10372.25	2	9635.14	-7.7%	-7.4%	-7.7%
R03	50	2	10402.10	2	10363.59	2	10356.25	2	9798.68	-6.2%	-5.8%	-5.7%
R04	50	2	10439.50	2	10332.77	2	10384.62	2	9735.61	-7.2%	-6.1%	-6.7%
R05	100	3	15659.83	3	15597.47	3	15618.67	4	19021.77	17.7%	18.0%	17.9%
R06	100	3	15695.33	3	15624.17	3	15618.71	4	18954.89	17.2%	17.6%	17.6%
R07	100	3	15703.10	3	15585.10	3	15602.87	4	19074.54	17.7%	18.3%	18.2%
R08	100	3	15640.82	3	15561.98	3	15556.00	4	19008.92	17.7%	18.1%	18.2%
R09	200	4	21086.26	4	21004.52	4	21108.37	7	32866.56	35.8%	36.1%	35.8%
R10	200	4	21101.61	4	21005.23	4	21099.53	7	32941.29	35.9%	36.2%	35.9%
R11	200	4	21006.22	4	21069.37	4	21089.72	7	32902.45	36.2%	36.0%	35.9%
R12	200	4	21079.60	4	21090.62	4	21100.88	7	32900.34	35.9%	35.9%	35.9%
RC01	50	2	10310.85	2	10296.97	2	10266.83	2	9605.45	-7.3%	-7.2%	-6.9%
RC02	50	2	10419.11	2	10323.85	2	10378.27	2	9711.33	-7.3%	-6.3%	-6.9%
RC03	50	2	10297.75	2	10250.24	2	10223.54	2	9613.95	-7.1%	-6.6%	-6.3%
RC04	50	2*	5836.50	2	10200.24	2*	5852.17	2	9523.62	38.7%	-7.1%	38.6%
RC05	100	3	15420.14	3	15454.60	3	15434.72	4	18790.52	17.9%	17.8%	17.9%
RC06	100	2	10728.72	3*	11145.33	2	10765.53	4	18826.16	43.0%	40.8%	42.8%
RC07	100	2	10653.71	3	15460.19	2	10723.67	4	18938.56	43.7%	18.4%	43.4%
RC08	100	3	15449.51	3	15418.75	3	15392.56	4	18791.35	17.8%	17.9%	18.1%
RC09	200	4	20991.80	4	21010.57	4	21085.41	7	32801.24	36.0%	35.9%	35.7%
RC10	200	4	20890.32	4	20909.28	4	20878.90	7	32592.93	35.9%	35.8%	35.9%
RC11	200	4	20836.77	4	20866.71	4	20840.60	7	32690.16	36.3%	36.2%	36.2%
RC12	200	4	20840.82	4	20832.99	4	20859.05	7	32617.13	36.1%	36.1%	36.0%

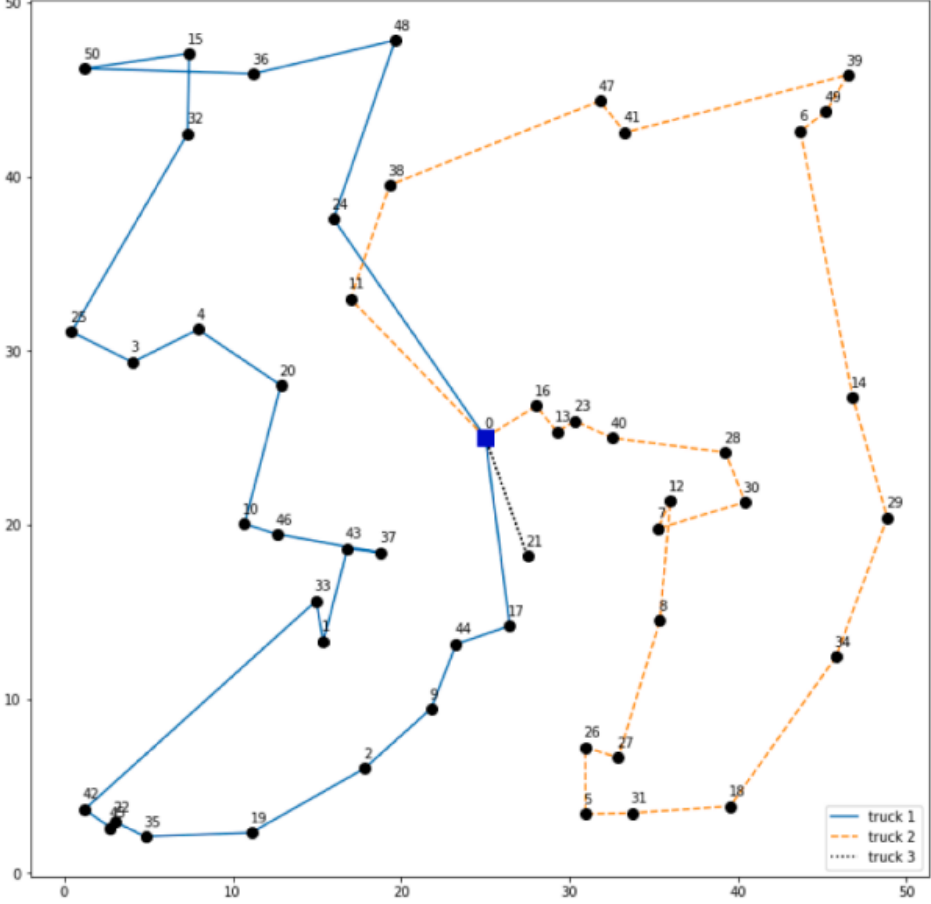
Note: NC: number of customers; SF: number of sub-fleets; NT: number of trucks; \*: one drone of 2nd sub-fleet is dispatched; \*\*: one drone from the 4th to the 9th sub-fleet is dispatched;  $\Delta_1$ : (FS Total Cost - VRP Total Cost)/(FS Total Cost);  $\Delta_2$ : (FS<sup>+</sup> Total Cost - VRP Total Cost)/(FS<sup>+</sup> Total Cost);  $\Delta_3$ : (FS<sup>+</sup>Total cost - VRP Total Cost)/(FS<sup>+</sup> Total Cost).

# 4. Computational tasks

## 4.4 Comparison of VRPD with VRP



a) VRPD (Total Cost: \$10,383.55)



b) VRP (Total Cost: \$14,175.99)

# 5. Conclusions

- The  $FS^{+1}$  mode performs better with instances from the C group,  $FS^+$  mode excels with instances from the R group, and the  $FS$  mode is more effective with instances from the RC group.
- Overall, the VRPD demonstrates cost savings of over 30% for large-scale instances when compared to the VRP, thus showing superior results.

Thank you