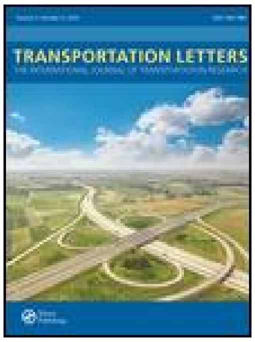


Transportation Letters

The International Journal of Transportation Research

ISSN: 1942-7867 (Print) 1942-7875 (Online) Journal homepage: https://www.tandfonline.com/loi/ytrl20

Lane changing models: a critical review

Sara Moridpour, Majid Sarvi & Geoff Rose

To cite this article: Sara Moridpour, Majid Sarvi & Geoff Rose (2010) Lane changing models: a critical review, Transportation Letters, 2:3, 157-173, DOI: 10.3328/TL.2010.02.03.157-173

To link to this article: https://doi.org/10.3328/TL.2010.02.03.157-173

Published online: 02 Dec 2013.

Sara Moridpour,[1] Majid Sarvi,[2] and Geoff Rose[2]*

Lane changing models: a critical review

ABSTRACT: Lane changing maneuvers have a significant impact on macroscopic and microscopic characteristics of traffic flows due to the interference effect they have on surrounding traffic. Understanding the factors which affect drivers' lane changing behavior is important due to the implication of lane changing models in variety of traffic and transportation studies. This paper reviews the existing lane changing models and assesses the strengths and weaknesses of each model type. In addition, the lane changing models are classified according to their characteristics. Then, limitations of the existing lane changing models are identified. Finally, the findings and conclusions of the paper are summarized and more promising research directions are suggested.

KEYWORDS: Lane changing models; Lane changing maneuvers; Drivers.

1. INTRODUCTION

Lane changing maneuvers have a fundamental impact on macroscopic and microscopic characteristics of traffic flows due to the interference effect they have on surrounding vehicles (Daganzo et al. 1999; Mauch and Cassidy 2002; Sasoh and Ohara 2002; Uddin and Ardekani 2002; Al-Kaisy and Hall 2003; Chen et al. 2004; Al-Kaisy and Jung 2005; Al-Kaisy et al. 2005, Sarvi and Kuwahara 2009).

One of the important effects of lane changing maneuvers on traffic flow characteristics could be speed and traffic flow oscillations. During heavy traffic conditions, the oscillation appears as a result of lane changing rather than car following (Mauch and Cassidy 2002; Laval and Daganzo 2006). When a vehicle changes lanes from one lane to another, this can have the effect of a capacity drop with shockwaves generated in both lanes (Sasoh and Ohara 2002; Jin 2010). Frequent lane changing maneuvers in merging, diverging and weaving areas can create bottleneck points in freeways and result in flow breakdown under heavy traffic conditions (Cassidy and Bertini 1999; Daganzo et al. 1999; Hoogendoorn and Bovy 2001; Daganzo 2002; Banks et al. 2003; Wall and Hounsell 2005). The flow breakdown which can arise from the lane changing maneuvers may potentially reduce the freeway safety (Wright 2006). Hence, developing an accurate lane changing model for drivers is an important component of model development.

*Corresponding Author

[1]School of Civil, Environmental and Chemical Engineering, RMIT University, Melbourne, VIC, Australia

[1]Department of Civil Engineering, Monash University, Melbourne, VIC, Australia

Lane changing models have application in a variety of traffic and transportation studies including transportation planning and development of traffic management policies (Adelakun and Cherry 2009; Yang and Regan 2009). To increase freeway capacity and traffic safety, numerous studies have examined different lane restriction strategies for different vehicle types including heavy vehicles (Cate and Urbanik II 2003; Chen et al. 2004; Al-Kaisy and Jung 2005; Adelakun and Cherry 2009; Yang and Regan 2009). Understanding the factors which influence drivers' lane changing behavior and developing capacities to model those decisions, has an important role to play in the development of traffic management strategies. Design and assessment of traffic policies is very difficult in real transportation networks due to the cost and risks of field trials. However, microscopic traffic simulation packages provide a virtual environment to evaluate new traffic management policies and measure their effects. Microscopic traffic simulation packages are capable of analyzing traffic behavior under different lane configurations, traffic compositions and traffic flow conditions (uninterrupted and interrupted). They have seen application in a variety of traffic and transportation studies. Due to the increasing reliance on microscopic traffic simulation software, it is important to improve their accuracy in modeling drivers' decisions. One of the essential components of any microscopic traffic simulation softwares is lane changing models. Therefore, it is important to ensure that the lane changing behavior of drivers is accurately captured in these models.

This paper reviews the existing lane changing models and assesses their strengths and weaknesses. In Section 2, existing lane changing models are classified according to their characteristics and application. A distinction is drawn

DOI 10.3328/TL.2010.02.03.157-173

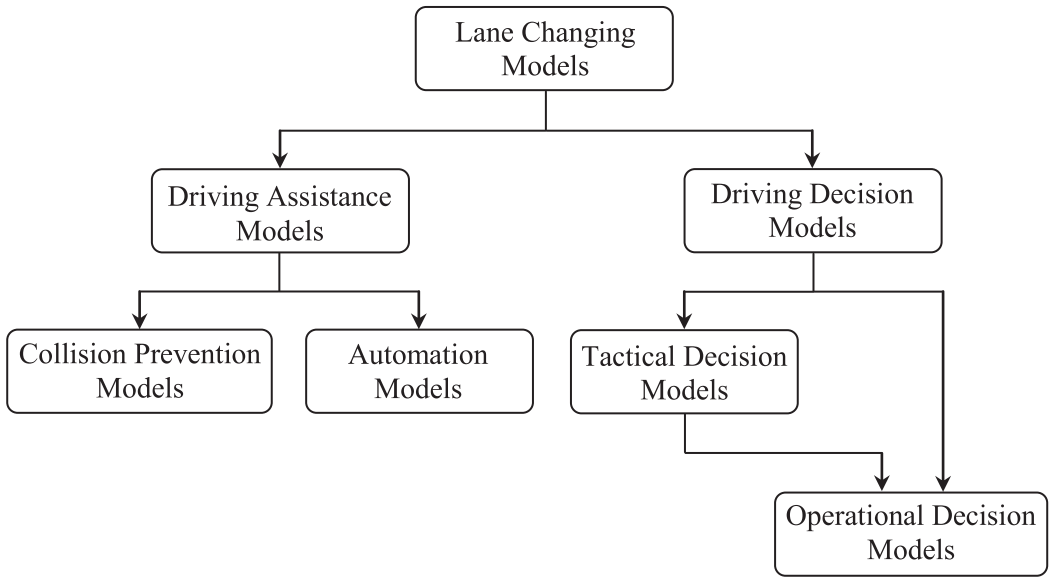


Figure 1. Classification of available approaches in lane changing studies.

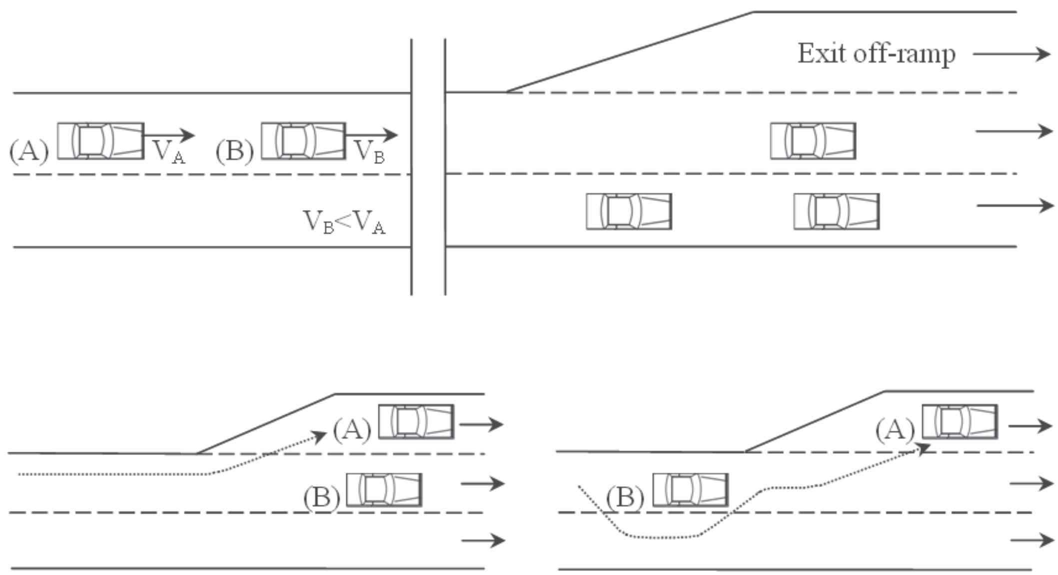
between 'Driving Assistance Models' and 'Driving Decision Models' with the latter being the primary focus of this paper. The 'driving Decision Models' are examined in detail in section 3 prior to an identification of the major limitations of the existing lane changing models in Section 4. Finally, section 5 summarizes the finding and conclusions of the paper and provides suggestions for further model development.

2. A CLASSIFICATION SCHEME FOR LANE CHANGING MODELS

Different approaches taken in lane changing studies can be classified as shown in Figure 1. In recent years, many studies have been focused on the scope for driving assistance systems to enhance road capacity and road safety (Lygeros et al. 1998; Nagel et al. 1998; Knospe et al. 2002; Hatipoglu et al. 2003; Mar and Lin 2005).

Driving assistance models can be classified as either collision prevention models or automation models. Both of these model types consider the steering wheel angle and lateral motions to control the lane changing performance of vehicles. Collision prevention lane changing models are developed to control drivers' lane changing maneuvers and assist them to execute a safe lane change. The collision prevention models are intended to improve road safety. Automation models are applied to perform the driving tasks either partially or entirely. Many different configurations are defined for models in this category such as lane change or side crash avoidance systems. Those applications involve automotive adjustments to the steering wheel angle of vehicles to control their lateral motion and reduce dangerous lane changing maneuvers (Lygeros et al. 1998; Nagel et al. 1998; Maerivoet and Moor 2005; Eidehall et al. 2007; Salvucci and Mandalia 2007; Doshi and Trivedi 2008; Kiefer and Hankey 2008; Li-sheng et al. 2009).

The other category of lane changing models focuses on drivers' lane changing decisions under different traffic conditions and under different situational and environmental characteristics. While responding to the surrounding environment, drivers' decisions can be classified as either strategic, tactical and operational (Sukthankar et al. 1997). This classification is based on the required time for executing the decisions. The strategic level is the highest decision level and deals with drivers' decisions which require over 30 seconds making and executing. Strategic level decisions are usually made before the start of trip and include the goal or purpose of the trip and choice of route. A driver's destination choice, mode choice and route choice are examples of strategic driving decisions (Alexiadis et al. 2004). While executing the strategic level decisions, a series of tactical decisions are made by the drivers. At the tactical level, maneuvers are selected to achieve short term objectives such as a decision to pass a slow moving vehicle or maintaining the desired speed. At the tactical, or intermediate, decision level, the time required for making and executing the decisions is between 5 and 30 seconds (Alexiadis et al. 2004). At the lowest decision level or the operational level, the maneuvers are converted to control operations. At this decision level, drivers decide about the maneuvers to control their vehicles. These take place on a



(a) Response 1: Remaining behind the slow moving vehicle and taking the exit off-ramp.

(b) Response 2: Passing the slow moving vehicle and taking the exit off-ramp.

Figure 2. A simple illustration for the tactical lane changing decision.

time scale of less than five seconds and include decisions such as whether or not to accept a gap (Alexiadis et al. 2004).

Lane changing decision models can be categorized as either tactical or operational since they are not made at the strategic level. In tactical lane changing decision models, drivers make their lane changing decisions based on current and anticipated future characteristics of the surrounding traffic. In operational lane changing decision models, the drivers' lane changing decision only depends on limited information on the current characteristics of the surrounding traffic. In operational lane changing decision models, drivers make very short term anticipations to decide about their lane changing maneuvers (El Hadouaj et al. 2000). In real traffic situations, drivers make tactical and operational decisions for their lane changing maneuvers based on the current characteristics of the surrounding traffic and their anticipated future characteristics of the surrounding traffic (El Hadouaj et al. 2000).

A simple illustration of tactical lane changing decision of a driver is depicted in Figure 2. In this figure, the driver of the subject vehicle (vehicle A) decides to take the exit off-ramp. Decision to execute a lane changing maneuver to take the exit off-ramp is an operational lane changing decision. However, the driver is obstructed by a slow moving vehicle (vehicle B).

In this example, depending on the distance to the exit off-ramp, the traffic condition and also the characteristics of the driver and vehicle, the driver may have two different responses. The first response is remaining in the lane which is nearest to the shoulder and accepting the speed limitation caused by the slow moving vehicle in order to remain in the correct lane to take the exit off-ramp (Figure 2.a). The alternative is executing a lane changing maneuver to pass the slow moving vehicle and then taking the exit off-ramp (Figure 2.b).

Lane changing decision models underpin the microscopic traffic simulation packages being applied increasingly in research and practice. These models are the primary focus of this review and are considered in greater detail in the following section.

3. DRIVING DECISION MODELS

In exploring the range of lane changing decision models, we adopt the conventional distinction between tactical and operational lane changing decisions. We define a two type classification of lane changing decision models: those that use an explicit search process to estimate the future position of drivers and those that look for associations with surrounding traffic characteristics. In the first case, it is necessary to estimate the position of vehicles in the near future as well as to estimate the drivers' lane changing decision as a result of the choices they face. In this way, a driver's decision to execute a lane changing maneuver (the operational lane changing decision) is based on traffic characteristic of the surrounding vehicles. However, the drivers' tactical lane changing decision is according to estimated results of search algorithms. In the other category of model, those based on traffic characteristics, the drivers' tactical and operational lane changing decisions are estimated according to characteristics of the surrounding traffic. A number of lane changing decision models which are based on either an explicit search algorithm or correlation with surrounding traffic characteristics, are reviewed separately in the subsections which follow.

3.1. Lane changing decision models based on a search algorithm

The concept of tactical decisions has seen application in computer science and robotics. Simulating drivers' decisions is similar to designing reactive robots. In contrast to robotics, where the robot's objective is to find an optimal solution for the problem it faces, the drivers decisions do not usually correspond to optimal solutions (Sukthankar et al's 1997). This is due to the fact that the drivers' decisions are based on imprecise perceptions and information.

In lane changing decision models based on search algorithm, the drivers' lane changing execution is the result of their tactical and operational lane changing decisions. In this model type, drivers' tactical lane changing decision is based on search algorithms while the drivers' operational lane changing decisions are based on surrounding traffic characteristics. The search algorithms which are originally used in computer science and robotics are used to estimate the drivers' lane changing decision and therefore, the near future position of vehicles.

Schlenoff et al. (2006) developed PRediction In Dynamic Environment (PRIDE), which is a hierarchical framework for moving object prediction which incorporates multiple prediction algorithms into a single framework. They tried to develop a framework in which the results from a short term prediction algorithm can be used to strengthen or weaken the results of a situation based long term prediction algorithm.

In Schlenoff et al long term prediction algorithm, each vehicle's current position and speed is used as an input to the algorithm. For each possible future action (e.g. acceleration, changing lanes), the algorithm creates a set of next possible positions and allocates a cost to each action. The cost reflects the danger to which a driver would be exposed by performing an action with a higher cost reflecting greater danger. The cost is calculated based on traffic characteristics of the surrounding vehicles and the distance of the vehicle from surrounding obstacles. The total cost is assumed to be the sum of all costs associated with performing each action. According to the total cost of each action, the algorithm computes the probability of performing that action by the driver. The algorithm also predicts trajectories of each vehicle, based on the possible path which the vehicle will take in predetermined time intervals. Then, the algorithm recalculates the vehicle's position set and the probability that the vehicle will be placed in each position. Finally, the future position of each vehicle is determined according to the highest probability of the locations. To combine the results of the short term and long term prediction algorithms, they developed a new methodology. For each vehicle, a set of positions and associated probabilities are determined and the distance between the positions obtained from the two algorithms is computed. If the distance is less than a threshold, no adjustment is made and the most probable position from the long term prediction algorithm is then used. If the distance is more than the threshold, the distance between the positions predicted by the short term and long term prediction algorithms is calculated. Then, the position with the least distance is accepted as the next position and all other probabilities are adjusted accordingly.

Schlenoff et al. graphically compared the estimated positions with the observed positions from actual field data to assess the accuracy of the short term and long term prediction algorithms. They reported that the short term prediction algorithms predict the future position of the vehicles with the minimum difference to the observed positions when focused on a time horizon of two seconds. Long term prediction algorithms were found to predict the future position of the vehicles with the minimum difference to the observed positions for a prediction horizon up to ten seconds.

Schlenoff et al. compared the results obtained from the short term and long term prediction algorithms to replicate the decisions of the subject vehicle driver and the drivers of surrounding vehicles at the same time. However, they considered only the current characteristics of vehicles (e.g. positions, speeds) to estimate their future positions and did not consider each driver's perception of other drivers' probable decisions. They assumed that none of the surrounding vehicles execute any lane changing maneuvers. In addition, they considered the interaction of vehicles only by the probability of a collision between them.

Webster et al. (2007) developed a lane changing decision model based on a forward search algorithm. The forward search algorithm generates a branching tree of sequential actions for each modeled vehicle at each time step. This algorithm considers the changes in the states of the subject vehicle and the surrounding vehicles to generate the branching tree of sequential actions. Each branch represents a particular action which is selected by the driver and the events which would probably have occurred as a result of this action. Finally, the sequence of actions leading to the best outcome, the minimum travel time, is selected and the subject vehicle driver applies the first action of that sequence.

Webster et al. compared their lane changing decision model to a basic lane changing decision model based on traffic characteristics, reflecting Gipps's lane changing structure (Gipps 1986). Gipps's lane changing model will be explained in the following section. In the basic lane changing decision model, if a lane change is possible, then the target lane is selected. The target lane is selected as the lane with the greatest allowable speed. The final stage is acceptance of a sufficient size gap for the maneuver. The drivers verify the gap size in the target lane before changing lanes and check the speeds of the vehicles at either end of the gap to ensure the gap is of sufficient size. The lane changing decision model developed by Webster et al. uses the same gap acceptance criteria as applied in the basic lane changing decision model. In their lane changing decision model, the lane changing decision is made using the information from the forward search tree, which is constructed at each time step that the subject vehicle has an available gap in either adjacent lane. The forward search tree is built starting at an initial time step, considering the speeds and positions of the subject vehicle and all surrounding vehicles upstream or downstream of the subject vehicle within a view distance specified as a model parameter. In the forward search algorithm, the subject vehicle is represented at its original position and speed at the initial time step, and then control is turned over to the driver decision model. In addition, the surrounding vehicles are represented as they actually travel and based on vehicle trajectory data.

Webster et al. calibrated their lane changing decision model using real vehicle trajectory data. To calibrate the model, they minimized the number of simulated lane changing maneuvers which did not occur in real traffic (Equation 1).

$$U_{LC} = \frac{1}{N}\sum_{i=1}^{N} \delta_i \qquad (1)$$

Where, U_{LC} is the lane change model performance index, i is the number of time steps over the duration of the simulation, N is the total number of time steps and $\delta_i = 0$ if the estimated lane changing maneuver (left, right, or no lane change) equals observed lane changing maneuver at time step i and 1 otherwise.

To test the accuracy of their lane changing model, they used observed trajectory dataset. They simulated 70 vehicles and calculated the lane change model performance index (U_{LC}), for each individual vehicle. The mean value of the ULC for the basic lane changing model and the Webster's model were 0.045 and 0.040 respectively. The results showed that for 22 simulated vehicles, the lane change performance index of Webster's model was smaller than the basic lane changing model which indicates the better performance of Webster's model. For 10 simulated vehicles, the lane change performance index of the basic lane changing model was smaller than Webster's model and for 38 simulated vehicles the performance index was similar for both models.

Webster et al's lane changing decision model relies on a number of simplifying assumptions. To construct the forward search tree, the surrounding vehicles are assumed not to change their acceleration/deceleration or perform any lane changing maneuvers. This assumption is in contrast to the real traffic situation in which the decisions of the subject vehicle driver affect the decisions of the surrounding vehicles' drivers. The lane changing decisions are also restricted to situations where an acceptable gap is available in the adjacent lanes. This assumption is only acceptable under free flow conditions. To provide acceptable gaps in heavy traffic conditions, the subject vehicle driver may force the lag vehicle driver in the target lane to slow down or the target lag vehicle driver may provide courtesy to the subject vehicle and create a gap. Therefore, subject vehicle driver can undertake a lane change.

3.2. Lane changing decision models based on traffic characteristics

In lane changing decision models in this category, drivers' tactical and operational lane changing decisions are based on surrounding traffic characteristics. In a tactical lane changing decision, drivers make their lane changing decisions based on current and anticipated future characteristics of the surrounding traffic. In an operational lane changing maneuver, the driver considers the current traffic situation to execute a lane changing maneuver.

Many studies have related the lane changing decision of drivers to surrounding traffic characteristics and they have produced models which fall into one of two broad categories: rigid mechanistic models and artificial intelligence models. Figure 3 highlights that there are a number of alternative models in each category.

Before explaining the different models in detail, it is appropriate to begin by explaining some key concepts and parameters which are common to all these models. These parameters include: positions, speeds and accelerations/

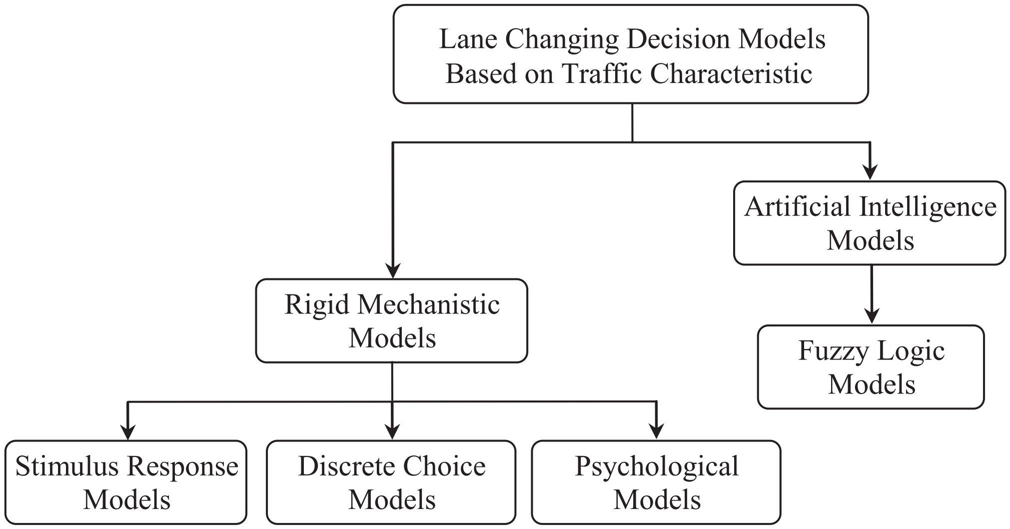


Figure 3. Classification of lane changing decision models based on traffic characteristics.

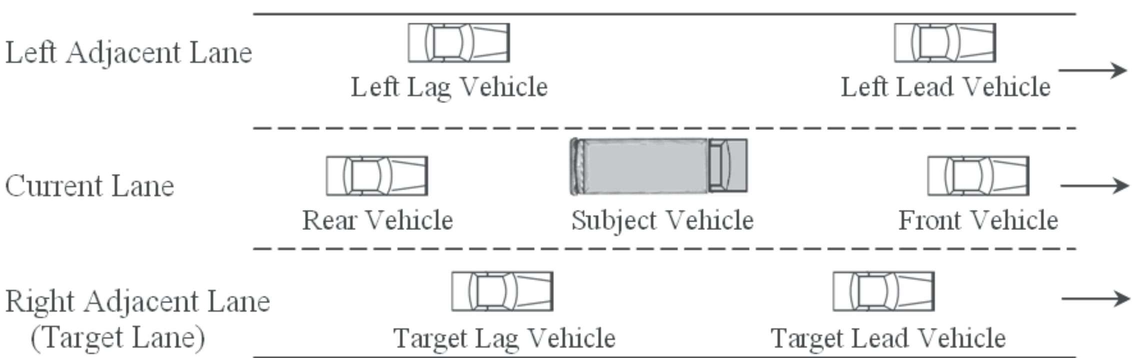


Figure 4. Spatial separation of the 'Subject Vehicle' from other vehicles in the traffic stream.

decelerations of the 'subject vehicle' and surrounding vehicles and the space gaps and relative speeds of the surrounding vehicles respect to the subject vehicle. Figure 4 shows the relationship between the subject vehicle and its surrounding vehicles for a case where the right adjacent lane is selected as the target lane.

3.2.1. Rigid mechanistic models

The rigid mechanistic models are those which create a crisp relationship between explanatory variables and dependant variable. In these models the magnitude of the result depends on the exact values of the independent variables. Mechanistic lane changing approaches do not usually incorporate the uncertainties associated with drivers' perceptions and decisions.

3.2.1.1. Stimulus response models

Gipps (1986) proposed a framework for the structure of lane changing decisions and the execution of lane changing. This framework is useful in explaining lane changing decisions on freeways and also on urban streets where traffic signals, obstructions and heavy vehicles influence drivers' decisions. In Gipps's model, the drivers' decision to execute a lane changing maneuver is the result of considering three factors: whether it is physically possible and safe to change lanes, whether it is necessary to change lanes and whether it is desirable to change lanes. He defined three zones to characterize the drivers' decisions during the lane changing maneuver. These three zones are based on the distance to the intended exit point. When the exit point is far away, it has no effect on drivers' lane changing decision and the drivers try to maintain their desired speed. When the exit point is a middle distance away, the drivers ignore the opportunities which have speed advantage but require changing lanes away from their desired exit point. When the drivers come close to their exit point, they should be in the correct lane or the adjacent lane and gaining a speed advantage is unimportant. These three zone areas represent a simplified tactical lane changing decision in Gipps's framework.

Gipps's lane changing decision model was developed on the basis of his car following model. The Gipps's car following model (Gipps 1981) imposes some limitations on driver's braking rate to have a safe speed with respect to the preceding vehicle. The driver's desired speed and the safe speed are considered at the same time in order to prevent the influence of the slow moving vehicles or obstructions which are far from the vehicle. Although Gipps proposed his lane changing structure based on his car following model, this lane changing structure can be applied using other car following models. The Gipps's car following model is given by the following equation.

$$v_n(t+T) = b_nT + [b_n^2T^2 - b_n(2(x_{n-1}(t) - s_{n-1} - x_n(t)) - v_n(t)T - v_{n-1}(t)^2/b_{n-1}^{est}]^{1/2} \tag{2}$$

Where, $v_n(t+T)$ is the maximum safe speed for vehicle n with respect to the preceding vehicle at time $(t+T)$, $b_n < 0$ is the maximum braking rate, T is the time between consecutive calculations of speed and position, $x_n(t)$ is the location of the front of vehicle n at time t, s_{n-1} is the effective length of vehicle $n-1$ which is the physical length plus a margin into which the subject vehicle is not willing to intrude and b_{n-1}^{est} is the estimate of b_{n-1} employed by the driver of vehicle n. For the purposes of the lane changing model, the maximum safe speed in Equation 2 is limited by the driver's desired speed and maximum braking. The driver's desired speed is used to prevent vehicles or obstructions too far ahead from influencing the driver's decision.

Gipps's lane changing decision model has never been validated using microscopic traffic and driver behavior data. The lane changing framework has been tested under various combinations of traffic conditions and the mistakes which had been found during the tests were adjusted. Gipps's lane changing decision model has been applied in several microscopic traffic simulations. Despite, its popularity, it is based on some simplifying assumptions. The lane changing occurs when a gap of sufficient length is available and it is safe to change lanes. However, this assumption may cause some limitations in heavy traffic conditions where appropriate gaps are rarely available and they are created when the lag vehicle drivers provide courtesy or when the subject vehicle drivers force the target lag vehicle drivers to reduce speed. In addition, the three zones in Gipps's model are defined deterministically based on the distance to the intended exit point and the differences between drivers and the differences within drivers over time are not considered.

Wiedemann and Reiter (1992) developed a theoretical lane changing decision model to explain the human decision process during the lane changing maneuver which is influenced by the driver's perception of surrounding vehicles. In this model, different drivers have different driving characteristics. These differences are observed in driving capabilities, abilities in perception and estimation of the surrounding traffic, safety requirements, desired speed and maximum acceptable acceleration/deceleration.

Wiedemann and Reiter assumed that the drivers' lane choice is influenced by their own wishes about driving. Based on this assumption, the model distinguishes between the lane changes from the slower to the faster lanes and the lane changes from the faster to the slower lanes. The desire to move into the faster lane can be due to an obstruction by a slow moving vehicle in the current lane. The level of obstruction is a function of the differences between the front vehicle speed and the subject vehicle driver's desired speed. A change to a faster lane is acceptable only when the level of obstruction in the current lane is higher than a specific threshold or if the speed of the lead vehicle in the target lane is considerably higher than the subject vehicle speed. The decision to move into the slower lane could be due to route choice or to allow a faster vehicle to pass. A change to a slower lane is accepted only when the subject vehicle will not be obstructed by a slow moving vehicle for a specific time interval. The tactical and operational decisions of drivers were considered in Wiedemann's lane changing decision model. Assuming that all drivers' decisions are based on human perceptions, they classified the surrounding influences as actual influences and potential influences. Actual influences are the real characteristics of surrounding vehicles which influence the driver's perceptions and decisions such as distances and relative speeds. Potential influences are the driver's estimation of the surrounding vehicles' situations in the near future.

Wiedemann and Reiter used the macroscopic characteristics of traffic flows to validate their lane changing decision model. They compared the estimated 'lane occupancy' for each lane as a function of traffic volume and traffic density with the corresponding values from a field dataset. They reported that their lane changing model resulted in a good fit of observed and estimated values of lane occupancy. They did not provide any quantitative indication to interpret the estimated results. They developed a general lane changing decision model for all vehicle types. Therefore, the parameters of the general lane changing decision model could be calibrated for the heavy vehicle drivers.

Hidas (2002; 2005) developed a lane changing decision model which considers the courtesy of the lag vehicle driver in the target lane during the merging or lane changing maneuver. Considering the different interaction types among the subject vehicle driver and the target lag vehicle driver, Hidas defined a three type classification of lane changing decisions: free, forced and cooperative lane changes. In free lane changes, there is no observable change in the gap between the lead and lag vehicles in the target lane during the lane changing maneuver. In forced lane changes, the gap between the lead and lag vehicles decreases before the start of the lane changing and increases after that. Cooperative lane changes have a reverse pattern with respect to the forced lane changes.

In addition, Hidas presented his forced lane changing algorithm based on the concept of drivers' courtesy. He assumed that a driver who wants to execute lane changing maneuver sends a courtesy request to subsequent vehicle drivers in the target lane. The request is evaluated by each lag vehicle driver in the target lane and depending on several factors such as position, speed and driver type of the lag vehicle it is either refused or accepted. The driver who provides courtesy reduces the speed to prepare a sufficient gap for the lane changing vehicle. Hidas also considered cooperative lane changing as a combination of two decisions: the willingness of the lag vehicle driver in the target lane to slow down and the feasibility to slow down. Definition of this three type classification of lane changes implies that the drivers' tactical lane changing decisions as well as their operational lane changing decisions was considered in Hidas's model.

Hidas assumed that lane changing is feasible if there is a gap of sufficient size for the driver in the target lane. The driver can move into the target lane without forcing the vehicles in the target lane to slow down significantly. In addition, lane changing is feasible if the deceleration or acceleration required for the subject vehicle to move behind the target lead vehicle and the deceleration required for the target lag vehicle to allow the subject vehicle to move into the target lane is acceptable. According to his lane changing model, the subject vehicle can move into the target lane if the target lead and lag space gaps are not less than the minimum acceptable target lead and lag space gaps at the end of the lane changing maneuver. The target lead space gap and the target lag space gap in a free lane changing maneuver are calculated as follows:

$$\begin{aligned} g_l &= g_{0l} - (v_s\, b_s/2) + v_l \\ g_f &= g_{0f} - (v_f\, b_f/2) + (v_s\, b_s/2) \end{aligned} \tag{3}$$

Where, g_l and g_f are the target lead and lag space gaps respectively, g_{0l} and g_{0f} are the target lead and lag space gaps at the start of lane changing, v_s, v_l and v_f are the speeds of the subject vehicle and the target lead and lag vehicles respectively and b_s and b_f are the decelerations of the subject vehicle and the target lag vehicle.

In a cooperative lane changing maneuver, the target lead space gap is calculated by Equation 3. The lag space gap in the target lane and the minimum acceptable target lead and lag space gaps in a cooperative lane changing maneuver are calculated by Equations 4 and 5 respectively.

$$g_f = g_{0f} - (v_f\, D_t - b_f/2\, D_t^2) + v_s D_t \tag{4}$$

Where, $D_t = D_v/b_f$ is the time of the deceleration period (D_v = speed decrease of the subject vehicle).

$$g_{l,min} = g_{min} + \begin{cases} c_l(v_s - v_l) & \text{if } v_s > v_l \\ 0 & \text{otherwise} \end{cases} \quad g_{f,min} = g_{min} + \begin{cases} c_f(v_f - v_s) & \text{if } v_f > v_s \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Where, g_{min} is the minimum safe gap that is independent of the speed difference between vehicles (this may be taken as equal to the jam gap) and c_l and c_f are constants.

This condition is used by the lag vehicle driver in the target lane to decide whether or not to reduce speed and allow the subject vehicle to move into the target lane. Meanwhile, the subject vehicle driver evaluates the feasibility of performing a lane changing maneuver. Therefore, the minimum acceptable space gaps may be shorter than the desired space gaps for the given speeds. The procedure of a forced lane changing maneuver is identical to the cooperative lane changing maneuver. The only difference is that the subject vehicle driver makes assumptions about the maximum speed decrease, D_v, and the maximum deceleration, b_f, which the target lag vehicle driver will use in the given situation. If the maneuver is feasible with these assumed values, the subject vehicle driver will force the target lag vehicle driver to reduce speed and provide gap of sufficient size for lane changing execution.

Hidas implemented his lane changing decision model in ARTEMiS (Analysis of Road Traffic and Evaluation by Micro Simulation) and tested on several simple hypothetical road network scenarios to assess the adequacy of his model. Then, he examined the effects of his lane changing decision model on both macroscopic traffic characteristics and microscopic traffic characteristics (individual vehicles). He estimated the speed-flow relationship on a freeway section by his lane changing model and by the ARTEMiS default model and compared the results with a typical speed-flow curve calculated for the same traffic situation using the Highway Capacity Manual (HCM) 1994 method. The results showed that up to about 2000 veh/hr flow rate, the estimated speed-flow relationship by his model was close to the HCM curve. The estimated speeds were lower than what is expected from the HCM curve at traffic flow rates above 2000 veh/hr. While the average speed under heavy traffic conditions was around 60 km/hr, with Hidas's lane changing model, it dropped to about 50 km/hr and with ARTEMiS default model it was below 40 km/hr. However, Hidas's lane changing model was more accurate than the ARTEMiS default model in replicating the HCM curve. Then, he analyzed the estimated positions and speed profiles of the subject vehicle and the target lead and lag vehicles in the forced and cooperative lane changing maneuvers and assessed their consistency with theory. He showed that the positions and the speed profiles of the subject vehicle and the lead and the lag vehicles are consistent with the theory of forced and cooperative lane change.

The general procedure in developing the stimulus response lane changing decision models, the considered stages in the model developments and the explanatory variables which were used in the literature to develop this model type are summarized in Table 2. Furthermore, some strengths and weaknesses of the stimulus response lane changing decision models are briefly presented in this table.

3.2.1.2. Discrete choice models

Ahmed (1996; 1999) developed a probabilistic model to describe lane changing decisions, based on a discrete choice framework. He modeled the lane changing decision as a sequence of three stages: decision to consider a lane change, choice of the target lane and acceptance of a sufficient size gap in the target lane to execute the lane changing decision. He defined three categories of lane changing maneuvers: Mandatory Lane Changing (MLC), Discretionary Lane Changing (DLC) and forced merging. MLC happens when a driver is forced to leave the current lane for instance when merging onto the freeway from an on-ramp or taking an exit off-ramp. DLC is performed when the driver is not satisfied with the driving situation in the current lane and wishes to gain some speed advantage for instance when the driver is obstructed by a slow moving vehicle (Yang and Koutsopoulos 1996). Forced merging occurs in heavily congested traffic conditions, when a gap of sufficient size is 'created' by drivers to enable them to execute a lane changing maneuver. The explanatory variables in Ahmed's MLC model include: the remaining distance to the point at which lane changing maneuver must be completed, the number of lane changes required to reach the lane connected to the next link and the delay (time elapsed since the MLC conditions apply). In his DLC model, if the driver is not satisfied with driving conditions in the current lane, the adjacent lanes are compared to the current lane and the driver selects a target lane. The mathematical formulation of the discrete choice framework constitutes two different probability functions. These two functions are applied to evaluate the probability of decision to execute a lane changing maneuver. According to Ahmed's lane changing decision model, the probability that driver n performs MLC, DLC or forced merging (FM) at time t are given by Equation 6.

$$P_t(LC \mid v_n) = \frac{1}{1 + \exp\left(-X_n^{LC}(t)\beta^{LC} - \alpha^{LC} v_n\right)} \qquad (6)$$

$LC = MLC, DLC, FM$

Where, $P_t(LC \mid v_n)$ is the probability of executing MLC, DLC or FM for driver n at time t, $X_n^{LC}(t)$ is the vectors of explanatory variables affecting decision to change lanes, β^{LC} is the corresponding vector of parameters, v_n is the driver specific random term and α^{LC} is the parameter of v_n.

After decision about changing lanes, the gap of sufficient size is accepted to execute the lane changing maneuver. The gap acceptance model captures whether the available gaps are accepted. In Ahmed's gap acceptance model, drivers are assumed to consider only the adjacent gaps. The available gap will be accepted if the target lead and lag gaps are acceptable. Ahmed defined the critical lead and lag gaps which are the minimum acceptable gaps. The available target lead and lag gaps will be accepted if they are greater than their critical values. The critical lead and lag gaps for driver n at time t is presented below.

$$G_n^{cr,\, gap\, j}(t) = \exp\left(X_n^{cr,\, gap\, j}(t)\, \beta^{gap\, j} + \alpha^{gap\, j}\, v_n + \varepsilon_n^{gap\, j}(t)\right) \qquad (7)$$

$gap\ j$ = lead, lag

Where, $G_n^{cr,\, gap\, j}(t)$ is the critical lead and lag gaps for driver n at time t, $X_n^{cr,\, gap\, j}(t)$ is the vector of explanatory variables affecting the critical gap j, $\beta^{gap\, j}$ is the corresponding vector of parameters, v_n is the driver specific random term, $\alpha^{gap\, j}$ is the parameter of v_n and $\varepsilon_n^{gap\, j}(t) \sim N(0, \sigma_{\varepsilon j}^2)$ is random term.

The probability of accepting a gap in MLC, DLC or FM for driver n at time t is given by Equation 8 in which $G_n^{lead}(t)$ and G_n^{lag} (t) are the available lead and lag gaps in the target lane.

$$\begin{aligned} &P_n(gap\ acceptance \mid v_n) = \\ &P_n(lead\ gap\ acceptable \mid v_n) \times P_n(lag\ gap\ acceptable \mid v_n) = \\ &P_n(G_n^{lead}(t) > G_n^{cr,\, lead}(t) \mid v_n) \times P_n(G_n^{lag}(t) > G_n^{cr,\, lag}(t) \mid v_n) \end{aligned} \qquad (8)$$

Ahmed used MITSIM (MIcroscopic Traffic SIMulator) as a test platform to assess the accuracy of his lane changing model. MITSIM is a microscopic traffic simulation laboratory developed to evaluate Advanced Traffic Management Systems (ATMS) and Advanced Traveller Information Systems (ATIS) at the operational level. He implemented his model in MITSIM and graphically compared the traffic volumes and average speeds estimated by his model and

MITSIM default model with observations in the field data. He showed that the performance of the MITSIM improved when the lane changing model of the default MITSIM model was replaced by his lane changing model. However, there is no quantitative measurement to show the better performance of MITSIM after implementing Ahmed's lane changing model.

Considering the distance to the lane changing point and the required number of lane changes, represents a tactical as well as an operational lane changing decision in Ahmed's model. His lane changing decision model captures the differences between drivers' lane changing decisions in MLC, DLC and forced merging. However, this model is unable to capture the trade-off between MLC and DLC decision process. For instance, in his model the drivers are unable to overtake while mandatory considerations are active. Similar to Gipps's model, this one assumes that the existence of the MLC situation is determined based on the distance to the exit off-ramp. In addition, the differences between the lane changing decision of passenger car and heavy vehicle drivers is only considered by a dummy variable when the subject vehicle is a heavy vehicle. The dummy variable only captures the difference in the size of the acceptable gap between a passenger car and a heavy vehicle. This is a very coarse and simplistic way to account for the differences in operational characteristics of these two vehicle types.

Toledo (2003; 2009) developed an integrated probabilistic lane changing decision model which allows drivers to consider both MLC and DLC at the same time. He used a discrete choice framework to model drivers' tactical and operational lane changing decisions and developed a probabilistic lane changing decision model. The model was calibrated using maximum likelihood estimation techniques. In his model, the lane changing decision is considered to comprise two steps: first, choice of the target lane and second, the gap acceptance decision. Toledo used a four group classification of the explanatory variables underlying lane changing decisions: neighborhood variables (e.g. gaps, speeds), path plan variables (e.g. distance from the intended exit off-ramp), network knowledge and experience (e.g. avoiding the nearest lane to the shoulder) and driving style and driving capabilities. Similar to Ahmed's model, he defined a dummy variable to capture the differences between the lane changing decision of passenger car and heavy vehicle drivers. The heavy vehicle dummy variable captures the limitations in heavy vehicles' speed.

In Toledo's target lane choice model, the driver may choose to stay in the current lane or to move into either the right or the left adjacent lanes. The general form of the model for target lane choice is presented by Equation 9.

$$U_n^{lane\ i}(t) = X_n^{lane\ i}(t)\ \beta^{lane\ i} + \alpha^{lane\ i} v_n + \varepsilon_n^{lane\ i}(t) \tag{9}$$
$$lane\ i = CL,\ RL,\ LL$$

Where, $U_n^{lane\ i}(t)$ is the utility of lane i to driver n at time t, $X_n^{lane\ i}(t)$ is the vector of explanatory variables affecting the utilities of lane i, $\beta^{lane\ i}$ is the corresponding vector of parameters, $\varepsilon_n^{lane\ i}(t)$ is the random term associated with the lane utility, v_n is the driver specific random term.

The probability of selecting a specific lane for each driver is calculated by a logit model (Equation 10).

$$P_n(lane\ i_t \mid v_n) = \frac{exp\ [X_n^{lane\ i}(t)^{lane\ i} + {}^{lane\ i} v_n]}{\sum_{j \in I} exp\ [X_n^{lane\ i}(t)^{lane\ j} + {}^{lane\ h} v_n]} \tag{10}$$
$$lane\ i,\ lane\ j\ I = \{CL,\ RL,\ LL\}$$

Based on the target lane choice, the gap acceptance model captures the decision on selecting the target gap. The model assumes that if the adjacent gap in the target lane is acceptable, the driver will move into the target lane and does not consider any other gaps. In his model, the adjacent gap comprises two parts: the target lead gap and the target lag gap. The available target lead and lag gaps are compared to driver's corresponding critical gaps, which are the minimum acceptable gaps. The available target lead and lag gaps will be accepted if they are greater than the critical target lead and lag gaps. The general form of Toledo's critical gap model is presented by Equation 11. To ensure that the critical gaps are always positive, they are assumed to follow a lognormal distribution.

$$\text{In}\ (G_n^{gap\ j,\ TL,\ cr}(t)) = X_n^{gap\ j,\ TL}(t)\ \beta^{gap\ j} + \alpha^{gap\ j} v_n + \varepsilon_n^{gap\ j}(t) \tag{11}$$
$$gap\ j = lead,\ lag$$

Where, $G_n^{gap\ j,\ TL,\ cr}(t)$ is the critical gap g in the target lane measured in meters, $X_n^{gap\ j,\ TL}(t)$ is the vector of explanatory variables affecting the critical gap j, $\beta^{gap\ j}$ is the corresponding vector of parameters, $\varepsilon_n^{gap\ j}(t) \sim \text{N}(0, \sigma^2_{gap\ j})$ is the random term and $\alpha^{gap\ j}$ is the parameter of the driver specific random term v_n.

The probability of accepting the gap and executing a lane changing maneuver for each driver is given by the following equation (Equation 12). In this equation, $G_n^{lead,\ TL}(t)$ and $G_n^{lag,\ TL}(t)$ are the available lead and lag gaps in the target lane.

$$P_n(\text{change to the target lane} \mid TL_t, v_n) = \tag{12}$$
$$P_n(\text{accept lead gap} \mid TL_t, v_n) \times P_n\ (\text{accept lag gap} \mid TL_t, v_n) =$$
$$P_n(G_n^{lead,TL}(t) > G_n^{lead,TL,cr}(t) \mid TL_t, v_n) \times P_n(G_n^{lag,TL}(t) > G_n^{lag,TL,cr}(t) \mid TL_t, v_n)$$
$$TL = RL, LL$$

In Toledo's model, after selecting the target lane and finding a gap of sufficient size, the drivers perform a sequence of accelerations and decelerations in order to move into the target gap. In addition, a three stage model of acceleration behavior is used for the subject vehicle to select the target gap. First, an acceleration model is applied for the time that the subject vehicle driver wishes to stay in the current lane. Second, an acceleration model is applied when the subject vehicle executes a lane changing maneuver into a gap which is alongside the subject vehicle. Third, an acceleration model which is used when the subject vehicle accelerates or decelerates to move into the target gap which is not directly alongside the subject vehicle.

Like Ahmed (1999), Toledo implemented his integrated lane changing model in MITSIM and compared the estimated measurements with results of using separate MLC and DLC models in MITSIM and observations in the field data. To evaluate the accuracy of his lane changing model, travel times and the distribution of vehicles across lanes as well as the traffic volume and average speed at each 5 minutes time interval were estimated by his integrated model and MITSIM separate MLC and DLC models. Then, these results were compared to each other and to the corresponding values from the field data. The results showed that the difference between the observed and estimated travel times for Toledo's model and separate MLC and DLC models are 3.2% and 9.5% respectively. Both models had similar performance in replicating observed traffic volumes and the observed distribution of vehicles across lanes. Furthermore, the difference between the observed speed and the estimated speed by Toledo's model was -2.9%. This value was -5.6% for separate MLC and DLC models.

The general procedure to develop a probabilistic lane changing decision model, the considered stages in the model developments, the explanatory variables and the strengths and weaknesses of this model type are summarized in Table 2.

3.2.1.3. Psychological models

The French National Institute for Research in Transportation and Safety (INRETS) developed a driving decision model called ARCHISIM. This model has been developed to use in either a driving simulator or as part of an ordinary traffic simulation model (Espié et al. 1994; Champion et al. 2001; Champion et al. 2002; El Hadouaj et al. 2000). In psychological driving decision models, drivers try to minimize their interaction with the surrounding traffic. In ARCHISIM, the decisions of subject vehicle drivers are based on trying to minimize the interaction with their environment including other drivers and road characteristics. Within ARCHISIM, drivers are simulated in virtual vehicles and they have individual models of their environment and they interact with other vehicles (e.g. passenger cars, trucks and trams), the infrastructure (e.g. traffic lights) and the road. Each driver has specific skills, aims and characteristics. The English publication on ARCHISIM presents only a general overview of the model and do not present the details of the model formulation or validation results. Consequently, it is impossible to comment in detail about the structure of the model or to assess its performance.

3.2.2. Artificial Intelligence models (AI)

Rigid mechanistic models do not incorporate the inconsistencies and uncertainties of drivers' perception and decisions (McDonald et al. 1997). These models are based on crisp variable magnitudes (Das and Bowles 1999). Most of the traditional lane changing decision models use crisp mathematical equations and conventional logic rules to represent drivers' knowledge of the surrounding traffic and to model the drivers' lane changing decisions. Commonly, random terms are included in these models which capture the variation of the explanatory variables around the mean value of those variables. The random terms are mainly Gumbel or normally distributed (Ahmed 1999; Choudhury et al. 2007; Toledo 2009).

However, drivers make their decisions based on their imprecise perceptions of the surrounding traffic. In recent years, Artificial Intelligence (AI) based approaches have become popular because they overcome the shortcoming of rigid mechanistic models. One type of artificial intelligence is fuzzy logic models which allow defining uncertainty in the model and therefore, reflect the natural or subjective perception of real variables (Ma 2004).

Das et al. (1999) proposed a new microscopic simulation methodology based on fuzzy IF-THEN rules and called the software package as Autonomous Agent SIMulation Package (AASIM). The major motivation of using a fuzzy knowledge based approach to model drivers' decisions is that fuzzy models provide an effective means to change highly nonlinear systems into IF-THEN rules. In addition, fuzzy logic is well equipped to handle uncertainties in real world traffic situations. They classified the lane changing maneuvers as MLC and DLC. To decide when a MLC happens in the microscopic traffic simulation package, the MLC fuzzy rules consider the distance to the approaching exit or merge point and the number of lane changes which are required. When multiple lane changes are required, the probability of making a decision to change lanes increases. The DLC rules of AASIM reflect a binary decision (change lanes or not) which is based upon two explanatory variables. These two explanatory variables are the driver's speed satisfaction level, which is based on the drivers' recent speed history, and the level of congestion in the left or right adjacent lanes. In AASIM, no specific lane changing decision model was considered for each vehicle type.

The formulation of driver's speed satisfaction level and congestion level in the left or right adjacent lanes are presented as Equations 13 and 14 respectively. The MLC and the DLC frameworks in AASIM are based on the tactical and operational lane changing decisions of drivers.

$$\sigma_t = (1 - \varepsilon) \times \sigma_{t-1} + \varepsilon \times (\frac{v}{v_{lim}}) \qquad (13)$$

Where, σ is the driver satisfaction, v is the vehicle speed during the current iteration, v_{lim} is the speed limit of freeway and ε is learning satisfaction rate.

The driver satisfaction represents the drivers' recent speed history which is updated at each time step. The learning satisfaction rate remains close to unity when the vehicle speed is maintained close to the speed limit and it reduces at lower speeds.

$$c = \frac{\sum_{all\ i} e^{-d_i/\Delta} \times (1 - \frac{v}{v_{lim}})}{\sum_{all\ i} e^{-d_i/\Delta}} \qquad (14)$$

Where, c is the local lane congestion as seen from a driver's view point, d_i is the distance to the i^{th} vehicle and Δ is parameter.

The summation in Equation 14 is carried out for all vehicles ahead of a vehicle. In this equation, the quantity $(e^{-d_i/\Delta})$ is a weight associated with the ith vehicle which decreases exponentially with distance.

In AASIM, once the driver decides to execute a lane changing maneuver, the next step is to find a suitable gap. The fuzzy rules are based on the adjacent gaps and surrounding vehicles' speeds in the target lane. Then, an acceleration value is calculated which is different from that generated by normal car following rules. If there is an acceptable size of gap in the target lane, the gap finding rules enable the vehicle to speed up or slow down to move closer to the gap. At the same time, the gap finding rules consider the safe headway to the front vehicle in the current lane. The last stage in AASIM lane changing decision model is setting the gap acceptance rules. These rules look for the gaps and speeds of the lead and lag vehicles in the target lane and the distance to the next exit or lane merge (infinite for DLC). The general form of a fuzzy rule which is used in their research is presented below.

j^{th} rule: If I_1 is $\boldsymbol{A_{1j}}$ and ... I_i is $\boldsymbol{A_{ij}}$ and I_m is $\boldsymbol{A_{mj}}$ then O is $\boldsymbol{B_j}$ (15)

Where, $I = f(I_1, I_1, ..., I_n)$ are the input variables, A_{ij} are the fuzzy subsets for input I_j, O is the output and B_j is the fuzzy subsets for output O.

Table 1: Fuzzy sets for lane changing decision model to the slower lane.

Pressure From Rear	Gap Satisfaction	Intension of Moving into Right Lane
High	Good	High
Medium	Moderate	Medium
Low	Bad	Low

They evaluated the accuracy of AASIM using actual field data. The simulation results were compared with the results from a commercial microscopic traffic simulation package called CORSIM (CORridor SIMulation). The traffic volume and the average speed of the weaving section were estimated for 60 minutes with AASIM and CORSIM. Then, the estimated results were compared to each other and to the field data observations. The results showed the average speeds of the simulated vehicles by AASIM differed from the field observations by less than 4.8 km/hr for each 15 minute time interval. In contrast, the speed differences from CORSIM model were around 16.0 km/hr.

McDonald et al. (1997) Brackstone et al. (1998) and Wu et al. (2000; 2003) developed a fuzzy logic motorway simulation model (FLOWSIM) and established fuzzy sets and systems for the model. To model the drivers' tactical and operational lane changing decision, they classified lane changing maneuvers into two categories: lane changes to the slower lane and lane changes to the faster lane. Lane changes to the slower are mainly executed to prevent disturbing fast moving vehicles which approach from the rear. Lane changes to the faster lane are mainly executed with the aim of gaining speed advantages. Their lane changing decision model to the slower lane uses two variables: pressure from the rear and gap satisfaction in the slower lane. The pressure from the rear is the time headway of the rear vehicle and gap satisfaction is the period of time during which it will be possible for the subject vehicle driver to stay in the gap in the slower lane, without reducing speed. The fuzzy sets which are used to develop their lane changing decision model to the slower lane are presented in Table 1.

A typical fuzzy rule in the lane changing decision model to the slower lane is given by Equation 16.

If pressure from rear is ***Low*** and gap satisfacion is ***High*** then intention of moving into right lane is ***Medium*** (16)

To establish the lane changing decision model to the faster lane, they defined two variables: overtaking benefit and opportunity. The overtaking benefit is the speed gained when a lane changing maneuver to the faster lane is executed. The

Table 2: Alternative lane changing decision models based on traffic flow characteristics.

<table>
<tr><th colspan="3">Model Types</th></tr>
<tr><th>Stimulus Response</th><th>Probabilistic</th><th>Fuzzy Logic</th></tr>
<tr><th colspan="3">General Procedure for Model Development</th></tr>
<tr><td>• Decide on explanatory variables.
• Calibrate the models (Gipps 1986; Wiedemann and Reiter 1992).</td><td>• Decide on:
1. Independent stages.
2. Explanatory variables.
3. Probability functions.
• Calibrate the probabilistic functions (Ahmed 1999; Choudhury et al. 2007; Toledo 2009).</td><td>• Decide on:
1. Explanatory variables.
2. Fuzzy sets and membership function.
3. Rule sets.
• Calibrate the models (McDonald et al. 1997; Brackstone et al. 1998; Das et al. 1999; Wu et al. 2000; Wu et al. 2003).</td></tr>
<tr><th colspan="3">Stages in Model Development and Explanatory Variables (EV)</th></tr>
<tr><td>• Decide on MLC or DLC (Gipps 1986).
EV: Maximum subject vehicle's safe speed and brake, front gap, subject vehicle driver's estimation of front vehicle driver's brake.
• Decide on lane change to either faster or slower lane (Wiedemann and Reiter 1992).
EV: Lane change duration, time and distance headways to surrounding vehicles.</td><td>• Decide on changing lanes.
EV: MLC-Exit/merge distance, number of lane changes, DLC-Presence of heavy vehicle, front relative speed and deceleration (Ahmed 1999).
• Select the target lane.
EV: Subject vehicle speed, target lead and lag gaps and relative speeds, presence of heavy vehicle, tailgating, avoiding the right-most-lane, distance to the exit off-ramp (Ahmed 1999; Toledo 2009).
• Accept a gap.
EV: Target lead and lag relative speeds, distance between target lead and lag (Ahmed 1999; Toledo 2009).</td><td>• Decide on MLC or DLC (Das et al. 1999).
MLC or DLC
EV: MLC-Exit/merge distance, number of lane changes, DLC-Left and right lane density, drivers' satisfaction.
Find a gap in target lane.
EV: Front, lead and lag gaps and relative speeds.
Accept sufficient size gap.
EV: Target lead and lag speeds and gaps, exit/merge distance.
• Change lanes to left or right (McDonald et al. 1997; Brackstone et al. 1998; Wu et al. 2000; Wu et al. 2003).
EV: Left-Motivation, opportunity, Right-Pressure, Gap satisfaction.</td></tr>
<tr><th colspan="3">Strengths</th></tr>
<tr><td>• Simplicity in modeling the lane changing maneuver.
• Considering the whole lane change decision process in one simple stage.
• Small number of variables.</td><td>• Decide on the basis of maximum gained utility.
• Probabilistic results instead of binary answers (yes/no).</td><td>• Considering human's imprecise perception.
• Calibrating the model with an optimization algorithm.
• Finding the fuzzy rules from numerical data.</td></tr>
<tr><th colspan="3">Weaknesses</th></tr>
<tr><td>• Difficulties in calibrating the model parameters.
• Using primary variables and simple framework to model the lane changing decision.</td><td>• Obligation to calculate all probability functions to find the utility of each choice.</td><td>• Validation process of the membership functions.
• Difficulties and complexity in abstracting fuzzy rules.</td></tr>
</table>

opportunity reflects the safety and comfort of the lane changing maneuver, which is measured by the time headway to the first lag vehicle in the faster lane. The lane changing decision framework which is used in FLOWSIM is a general lane changing decision framework and could be applied for all types of vehicles. To assess the accuracy of FLOWSIM in estimating the lane changing maneuvers, the empirical 5 minutes traffic flow counts were used as the simulation inputs. They estimated the number of lane changing maneuvers and the percentage of lane occupancy for each lane at different traffic flow rates. The estimated results were then compared to the observations in the field data at two steps. First, they used the mean value of the measurements to compare the observed and the estimated measurements. They found the difference between the mean values of the observed and estimated lane changing rates and lane occupancy at different traffic flow rates.

The results showed that the differences between the observed and estimated measurements are in the range of 0-11%. To further analyze the results, they statistically tested and compared the distribution of the observed and estimated measurements showing that the overall distribution of the estimated and observed measurements also matches well.

The procedure to develop the fuzzy logic lane changing decision models, the explanatory variables and the strengths and the weaknesses of fuzzy logic lane changing decision models are summarized in Table 2.

4. LIMITATIONS OF THE EXISTING LANE CHANGING MODELS

From the foregoing review of the literature, the major limitations of the existing lane changing models become apparent. The literature review has highlighted a number of areas where further research could overcome gaps in existing knowledge which are presented below:

- The number of heavy vehicles on roadways of United States of America, has increased by 75% over the past three decades and this trend is likely to continue at least over the next decade (Bureau of Transportation Statistics 2002). Typically, the proportion of heavy vehicles ranges from as low as 2% to as high as 25% of total traffic during the day (Al-Kaisy et al. 2002). In Australia, the proportion of heavy vehicles could increase to 30% of total vehicles in the morning peak and 20% in the afternoon peak on some freeways (Conway 2005). However, little attention has been paid to a specific lane changing model for heavy vehicles. The current lane changing models are principally associated with passenger cars and do not explore or attempt to capture the differences which exist between the passenger car and heavy vehicle lane changing patterns. The current lane changing models mainly deal with calibrating the parameters of a general lane changing model for the heavy vehicles rather than considering a specific lane changing model for the heavy vehicle drivers (Moridpour et al. 2009; 2010).
- Several seconds are required for drivers to complete a lane changing maneuver. However, the existing lane changing models mainly focus on drivers' lane changing decision and generally neglect execution of the lane changing maneuver. Excluding the lane changing execution, may have a significant impact on estimated traffic flow characteristics. Considering the lane changing execution may improve the accuracy of lane changing models which subsequently could improve the accuracy of the obtained results from microscopic traffic simulations.
- The common approach in assessing the accuracy of the current lane changing models is to analyze the macroscopic traffic measurements estimated by the model and compare them to the observed values from field data. Evaluating the macroscopic traffic measurements is insufficient to test the performance of the lane changing models. To examine the accuracy of the lane changing models in further detail, the estimated lane changing maneuvers should microscopically be analyzed and compared to the observed lane changes in the field data. To microscopically analyze the estimated lane changing maneuvers, the speeds of the subject vehicle and the surrounding vehicles (Figure 4) and the space gaps of the surrounding vehicles respect to the subject vehicle should be compared to corresponding values in the observed lane changing maneuvers.

5. CONCLUSIONS AND FUTURE DIRECTIONS

Lane changing maneuvers have a significant impact on macroscopic and microscopic characteristics of traffic flows due to their interfering effect on surrounding traffic. Understanding the explanatory variables which affect drivers' lane changing behavior is important since lane changing models are imbedded in the microscopic traffic simulation software which now has application in variety of traffic and transportation studies. Studies have investigated the drivers' lane changing behavior based on a range of approaches. This paper provided a review on the existing lane changing models and described their strengths and weaknesses. The lane changing models were classified in this paper according to their characteristics.

In exploring the range of lane changing models, a two type classification of lane changing decision models was defined: lane changing decision models based on a search algorithm and lane changing decision models based on traffic characteristics. In the lane changing decision models which are based on a search algorithm, drivers' operational lane changing decision is modeled based on the surrounding traffic characteristics and drivers' tactical lane changing decision is estimated according to the search algorithms. The search algorithms are used to estimate the near future position of vehicles as well as estimating the drivers' lane changing decision. In the lane changing decision models based on traffic characteristics, drivers' tactical and operational lane changing decisions are estimated according to characteristics of the surrounding traffic.

The lane changing decision models based on traffic characteristics fall into one of two broad categories: rigid mechanistic models and AI based models. The rigid mechanistic models create a crisp relationship between the explanatory variables and the dependent variables. These models do not usually incorporate the uncertainties associated with drivers' perceptions and decisions. The fuzzy logic models which are one type of AI models provide the opportunity to define uncertainty in the model and therefore, reflect the natural perception of explanatory variables.

The 3 key limitations of the existing lane changing decision models were identified. First, no specific lane changing model has been developed for the heavy vehicle drivers. The current lane changing models mainly deal with calibrating the parameters of a general lane changing model for the heavy vehicle drivers. Second, the existing lane changing models mainly focus on drivers' lane changing decision and generally neglect the lane changing execution. Finally, macroscopic traffic measurements are used to examine the accuracy of the current lane changing models. Further analysis is required to examine the performance of lane changing models in replicating the observed lane changing maneuvers microscopically.

Future research should focus on advancing microscopic traffic flow modeling by providing an enhanced capability for modeling the lane changing of drivers. Such a lane changing model would require capturing specifications of the heavy vehicle drivers' lane changing decision in addition to the physical characteristics of heavy vehicles (e.g. length, size) and their operational characteristics (e.g. acceleration, deceleration and maneuverability). Acceleration/deceleration models should be developed for different vehicle types during lane changing execution. These models will estimate the acceleration and deceleration behavior of drivers while changing lanes. To increase freeway capacity and improve traffic safety, different lane restriction strategies may be considered for heavy vehicles and passenger cars. Microscopic traffic simulation packages could be used to assess the effects of different heavy vehicle and passenger car lane restriction strategies. A lane changing decision model for heavy vehicle drivers as well as lane changing execution models for heavy vehicle and passenger cars drivers will increase the accuracy of microscopic traffic simulation packages in estimating heavy vehicle and passenger car drivers' lane changing behavior and enhance the performance of microscopic traffic simulation packages.

To develop new lane changing models and improve the accuracy of the current lane changing models, large trajectory datasets are required. Collection and compilation of a large trajectory dataset for model development is costly and time consuming. However, it would be desirable for model development to be based on a large sample data. A large trajectory dataset would enable future research to model drivers' behavior under different traffic conditions. Furthermore, the differences between drivers as well as the differences in the behavior of individual drivers over time would be captured.

REFERENCES

Adelakun, A. and Cherry, C. 2009. Exploring Truck Driver Perceptions and Preferences: Congestion and Conflict, Managed Lanes, and Tolls. *Proceeding of the 88th Transportation Research Board Annual Meeting*, Washington, DC, CD-ROM.

Ahmed, K. I., Ben-Akiva, M., Koutsopoulos, H. N. and Mishalani, R. G. 1996. Models of Freeway Lane Changing and Gap Acceptance Behavior. *Proceedings of the 13th International Symposium on the Theory of Traffic Flow and Transportation.*

Ahmed, K. I. 1999. *Modeling Drivers' Acceleration and Lane Changing Behavior.* Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Master of Science.

Alexiadis, V., Gettman and Hranac, R. 2004. *Next Generation Simulation (NGSIM): Identification and Prioritization of Core Algorithm Categories,* Department of Transportation Federal HighWay Administration (FHWA).

Al-Kaisy, A. F., Hall, F. L. and Reisman, E. S. 2002. Developing passenger car equivalents for heavy vehicles on freeways during queue discharge flow. *Transportation Research A* 36(8): 725–742.

Al-Kaisy, A. and Hall, F. 2003. Guidelines for Estimating Capacity at Freeway Reconstruction Zones. *Journal of Transportation Engineering* 129(5): 572-577.

Al-Kaisy, A. and Jung, Y. 2005. Examining the Effect of Heavy Vehicles on Traffic Flow during Congestion. *Proceedings of the Institute of Transportation Engineers (ITE) Annual Meeting*, Melbourne, Australia.

Al-Kaisy, A., Jung, Y. and Rakha, H. 2005. Developing Passenger Car Equivalency Factors for Heavy Vehicles during Congestion. *Journal of Transportation Engineering* 131(7): 514-523.

Banks, J. H., Amin, M. R., Cassidy, M. and Chung, K. 2003. *Validation of Daganzo's Behavioral Theory of Multi-lane Traffic Flow: Final Report.* Institute of Transportation Studies California Partners for Advanced Transit and Highways (PATH).

Brackstone, M., McDonald, M. and Wu, J. 1998. Lane Changing on the Motorway: Factors Affecting Its

Occurrence, and Their Implications. *9th International Conference on Road Transport Information and Control*, London, UK.

Bureau of Transportation Statistics. 2002. US Department of Transportation, *National Transportation Statistics, BTS02-08,* Washington DC, US Government Printing Office.

Cassidy, M. J. and Bertini, R. L. 1999. Observations at a Freeway Bottleneck. *International Symposium of Transportation and Traffic Theory*. Amsterdam, Netherlands: 107-124.

Cate, M. A. and Urbanik II, T. 2003. Another View of Truck Lane Restrictions. *Proceeding of the 82th Transportation Research Board Annual Meeting*. Washington, DC., CD-ROM.

Champion, A., Éspié, S. and Auberlet, J. M. 2001. Behavioral Road Traffic Simulation with ARCHISIM. *Summer Computer Simulation Conference*, Orlando, USA.

Champion, A., Zhang, M. Y., Auberlet, J. M. and Espié, S. 2002. Behavioral Simulation: Towards High-Density Network Traffic Studies. *International Conference on Traffic and Transportation Studies*, (Guilin China).

Chen, W. Y., Huang, D. W., Huang, W. N. and Huang, W. L. 2004. Traffic Flow on a 3-Lane Highway. *International Journal of Modern Physics B* 18 (31-32): 4161-4171.

Choudhury, C. F., Ben-Akiva, M. E., Toledo, T., Lee, G. and Rao, A. 2007. Modeling Cooperative Lane Changing and Forced Merging Behavior. *Proceeding of the 86th Transportation Research Board Annual Meeting*. Washington, DC., CD-ROM.

Conway, K. 2005. *Pacific Highway Upgrade–F3 to Raymond Terrace Route (Consultancy Report)*. Maunsell Australia Pty Ltd.

Daganzo, C. F. 2002. A Behavioral Theory of Multi-lane Traffic Flow, Part II: Merges and the Onset of Congestion. *Transportation Research B* 36: 159-169.

Daganzo, C. F., Cassidy, M. J. and Bertini, R. L. 1999. Possible Explanations of Phase Transitions in Highway Traffic. *Transportation Research A* 33(5): 365-379.

Das, S. and Bowles, B. A. 1999. Simulations of Highway Chaos Using Fuzzy Logic. *18th International Conference of the North American Fuzzy Information Processing Society (NAFIPS)*, New York, USA.

Das, S., Bowles, B. A., Houghland, C. R., Hunn, S. J. and Zhang, Y. 1999. Microscopic Simulations of Freeway Traffic Flow. *Proceedings of the Thirty-Second Annual Simulation Symposium IEEE Computer Society*, Washington, DC, USA.

Doshi, A. and Trivedi, M. 2008. A Comparative Exploration of Eye Gaze and Head Motion Cues for Lane Change Intent Prediction. *IEEE Intelligent Vehicles Symposium*, Eindhoven, Netherlands: 49-54.

Eidehall, A., Pohl, J., Gustafsson, F. and Ekmark, J. 2007. Toward Autonomous Collision Avoidance by Steering. *IEEE Transactions on Intelligent Transportation Systems*, 8(1): 84-94.

El Hadouaj, S., Espié, S. and Drogoul, A. 2000. To Combine Reactivity and Anticipation: The Case of Conflicts Resolution In a Simulated Road Traffic. *The Second International Workshop on Multi-Agent Based Simulation*, Boston, USA.

Espié, S., Saad, F., Schnetzler, B., Bourlier, F. and Djemame, N. 1994. Microscopic Traffic Simulation and Driver Behavior Modeling: The ARCHISIM Project. *The Strategic Highway Research Program and Traffic Safety on Two continents*, Lille, France.

Gipps, P. G. 1981. A Behavioral Car-Following Model for Computer Simulation. *Transportation Research B* 15(2): 105-111.

Gipps, P. G. 1986. A Model for the Structure of Lane-changing Decisions. *Transportation Research B* 20(5): 403-414.

Hatipoglu, C., Ozguner, U., Redmill, K. A. 2003. Automated Lane Change Controller Design. *IEEE transactions on intelligent transportation systems* 4 (1): 13-22.

Hidas, P., 2002. Modelling Lane Changing and Merging in Microscopic Traffic Simulation. *Transportation Research C* 10(5-6): 351-371.

Hidas, P. 2005. Modelling Vehicle Interactions in Microscopic Simulation of Merging and Weaving. *Transportation Research C* 13(1): 37–62.

Hoogendoorn, S. P. and Bovy, P. H. L. 2001. State-of-the-art of Vehicular Traffic Flow Modeling. *Special Issue on Road Traffic Modelling and Control of the Journal of Systems and Control Engineering*, 215(4): 283-303.

Jin, W. L. 2010. A Kinematic Wave Theory of Lane-changing Vehicular Traffic. *Transportation Research B* 44(8-9):1001-1021.

Knospe, W., Santen, L., Schadschneider, A. and Schreckenberg, M. 2002. A Realistic Two-lane Traffic Model for Highway Traffic. *Journal of Physics A: Mathematical and General* 35(15): 3369-3388.

Kiefer, R. J. and Hankey, J. M. 2008. Lane Change Behavior with a Side Blind Zone Alert System. *Accident Analysis and Prevention*, 40(2): 683-690.

Li-sheng, J., Wen-ping, F., Ying-nan, Z., Shuang-bin, Y. and Hai-jing, H. 2009. Research on Safety Lane Change Model of Driver Assistant Aystem on Highway. *IEEE Intelligent Vehicles Symposium*, Shaanxi, China.

Laval, J. A. and Daganzo, C. F. 2006. Lane Changing in Traffic Stream. *Transportation Research B* 40(3): 251-264.

Lygeros, J., Godbole, D. N., Sastry, S. 1998. Verified Hybrid Controllers for Automated Vehicles. *IEEE Transactions on Automatic Control* 43(4): 522-539.

Ma, X. 2004. Toward an Integrated Car-following and Lane-changing Model Based on Neural-fuzzy Approach. *Helsinki summer workshop*.

Maerivoet, S., Moor, B. D. 2005. Cellular Automata Models of Road Traffic. *Physics Report*s 419(1): 1-64.

Mar, J., Lin, H. T. 2005. The Car-following and Lane-changing Collision Prevention System Based on the Cascaded Fuzzy Inference System. *IEEE Transactions on Vehicular Technology* 54(3): 910-924.

Mauch, M. and Cassidy, M. J. 2002. Freeway Traffic Oscillations: Observations and Predictions. *Proceedings of the 15th International Symposium on Transportation and Traffic Theory*, Elsevier Science Ltd., Adelaide, Australia, 2002, 653-673

McDonald, M., Wu, J. and Brackstone, M. 1997. Development of a Fuzzy Logic Based Microscopic Motorway Simulation Mode. *Proceedings of the IEEE Conference on Intelligent Transportation Systems*, Boston, U.S.A.

Moridpour, S., Rose, G. and Sarvi, M. 2009. Modelling the heavy vehicle drivers' lane changing decision under congested traffic conditions. *Journal of Road and Transport Research*, 18(4): 49-57.

Moridpour, S., Rose, G. and Sarvi, M. 2010. The effect of surrounding traffic characteristics on lane changing behavior. *Journal of Transportation Engineering* (In Press, ISSN 1943-5436-online).

Nagel, K., Wolf, D. E., Wagner, P. and Simon, P. 1998. Two-lane Traffic Rules for Cellular Automata: A Systematic Approach. *Physical Review E* 58(2): 1425-1437.

Salvucci, D. D. and Mandalia, H. M. 2007. Lane-change detection using a computational driver model. *Human Factors and Ergonomics Society*, 49(3): 532-42.

Sarvi, M. and Kuwahara, M. 2009. Improving the efficiency of freight traffic at congested freeway merging sections. *Transportation Letters Journal*, Volume 1, Issue 4.

Sasoh, A., Ohara, T. 2002. Shock Wave Relation Containing Lane Change Source Term for Two-Lane Traffic Flow. *Journal of the Physical Society of Japan* 71(9): 2339-2347.

Schlenoff, C., Madhavan, R. and Kootbally, Z. 2006. PRIDE: A Hierarchical, Integrated Prediction Framework for Autonomous On-Road Driving. *Proceedings of the 2006 IEEE International Conference on Robotics and Automation*, Orlando, Florida.

Sukthankar, R., Baluja, S. and Hancock, J. 1997. Evolving an Intelligent Vehicle for Tactical Reasoning in Traffic. *Proceedings of the International Conference on Robotics and Automation*, New Mexico, 519 - 524.

Toledo, T. 2003. *Integrated Driving Behavior Modeling.* Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Doctor of Philosophy.

Toledo, T., Koutsopoulos, H. N. and Ben-Akiva, M. 2003. Modeling Integrated Lane-changing Behavior. *Proceeding of the 82nd Transportation Research Board Annual Meeting.* Washington, DC, Transportation Research Board.

Toledo, T., Koutsopoulos, H. N. and Ben-Akiva, M. 2009. Estimation of an Integrated Driving Behavior Model. *Transportation Research C*, 17(4): 365-380.

Uddin, M. S., Ardekani, S. A., 2002. An Observational Study of Lane Changing on Basic Freeway Segment. *Proceeding of the 81st Transportation Research Board Annual Meeting*, Washington, DC.

Wall, G. T. and Hounsell, N. B. 2005. Microscopic Modeling of Motorway Diverges. *European Journal of Transport and Infrastructure Research*, 5(3): 139-158.

Webster, N. A., Suzuki, T., Chung, E. and Kuwahara, M. 2007. Tactical Driver Lane Change Model Using Forward Search. *Proceeding of the 86th Transportation Research Board Annual Meeting*, Washington, DC.

Webster, N. A., Suzuki, T. and Kuwahara, M. 2007. Driver Model for Traffic Simulation, with Tactical Lane Changing Behavior. *Monthly journal of the Institute of Industrial Science*, University of Tokyo 59(3): 188-191.

Wiedemann, R. and Reiter, U. 1992. *Microscopic Traffic Simulation: the Simulation System Mission, Background and Actual State. CEC Project ICARUS (V1052), Final Report, vol. 2, Appendix A*, Brussels: CEC.

Wright, S. J. 2006. *Review of Urban Congestion Trends, Impacts and Solutions (Consultancy Report).* Traffic Management Systems for Australian Urban Freeways, Australian Road Research Board (ARRB) Consulting-Council of Australian Governments.

Wu, J., Brackstone, M. and McDonald, M. 2000. Fuzzy Sets and Systems for a Motorway Microscopic Simulation Model. *Fuzzy Sets and Systems, Special Issue on Fuzzy Sets in Traffic and Transport Systems* 116(1): 65-76.

Wu, J. Brackstone, M. and McDonald, M. 2003. The Validation of a Microscopic Simulation Model: A Methodological Case Study. *Transportation Research C* 11(6): 463–479.

Yang, C. H. and Regan, A. C. 2009. Prioritization of Potential Alternative Truck Management Strategies using the Analytical Hierarchy Process. *Proceeding of the 88th Transportation Research Board Annual Meeting*. Washington, DC, Transportation Research Board.

Yang, Q. and Koutsopoulos, H. N. 1996. A Microscopic Traffic Simulator for Evaluation of Dynamic Traffic Management Systems. *Transportation Research C* 4(3): 113-129.